Use of Point-Based Stock Data for Growth Projection

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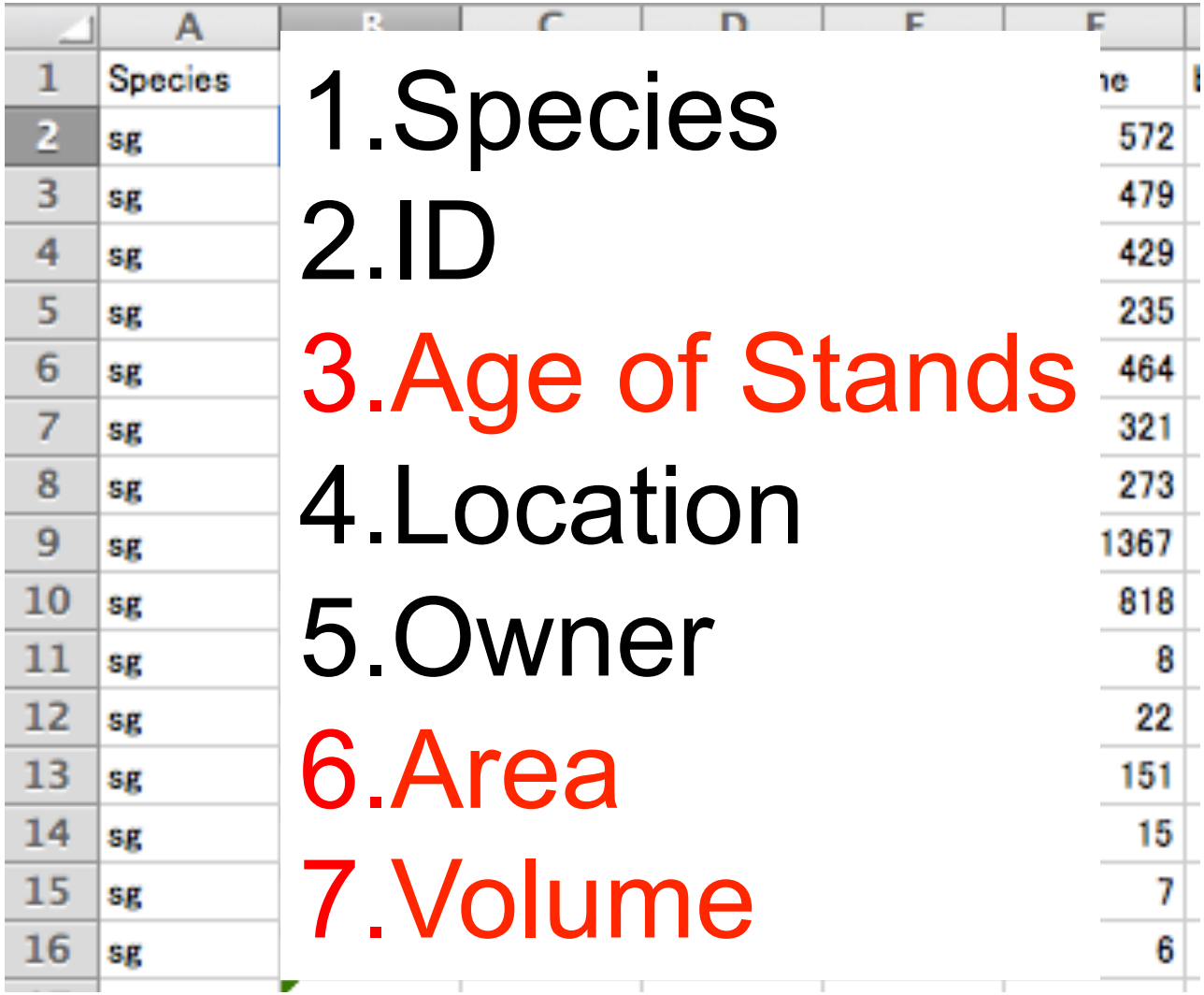
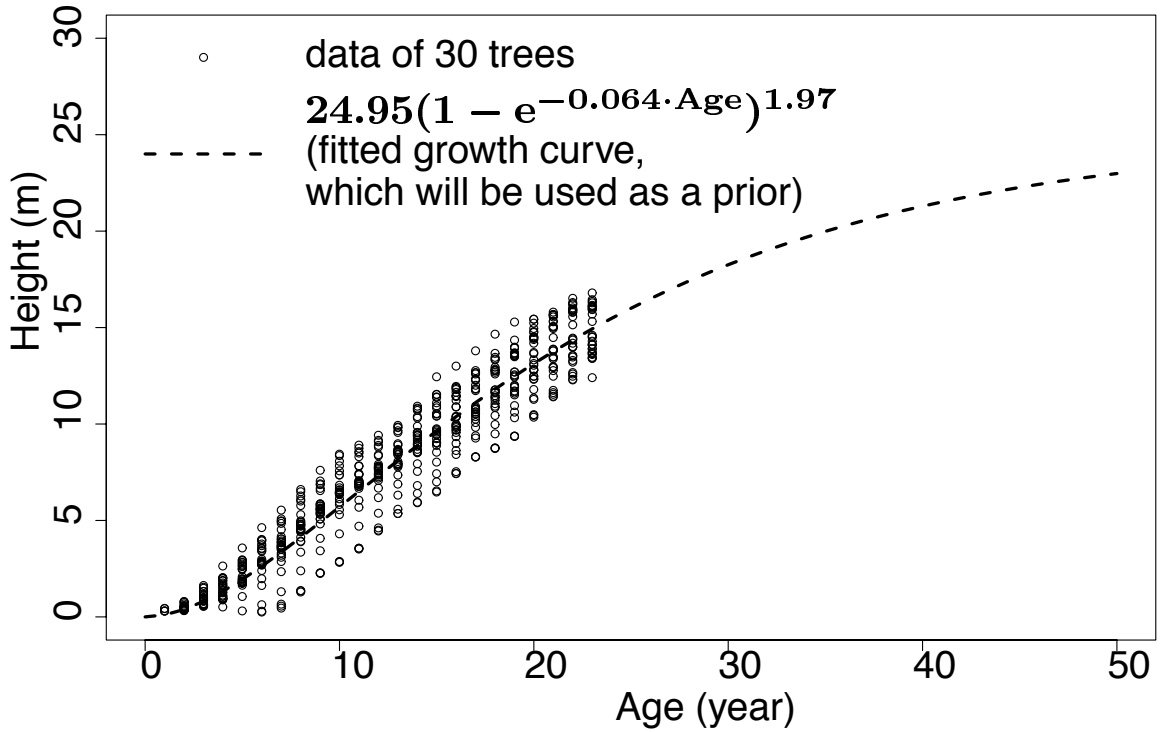
In Reality: Available Data or Information for Harvest Scheduling

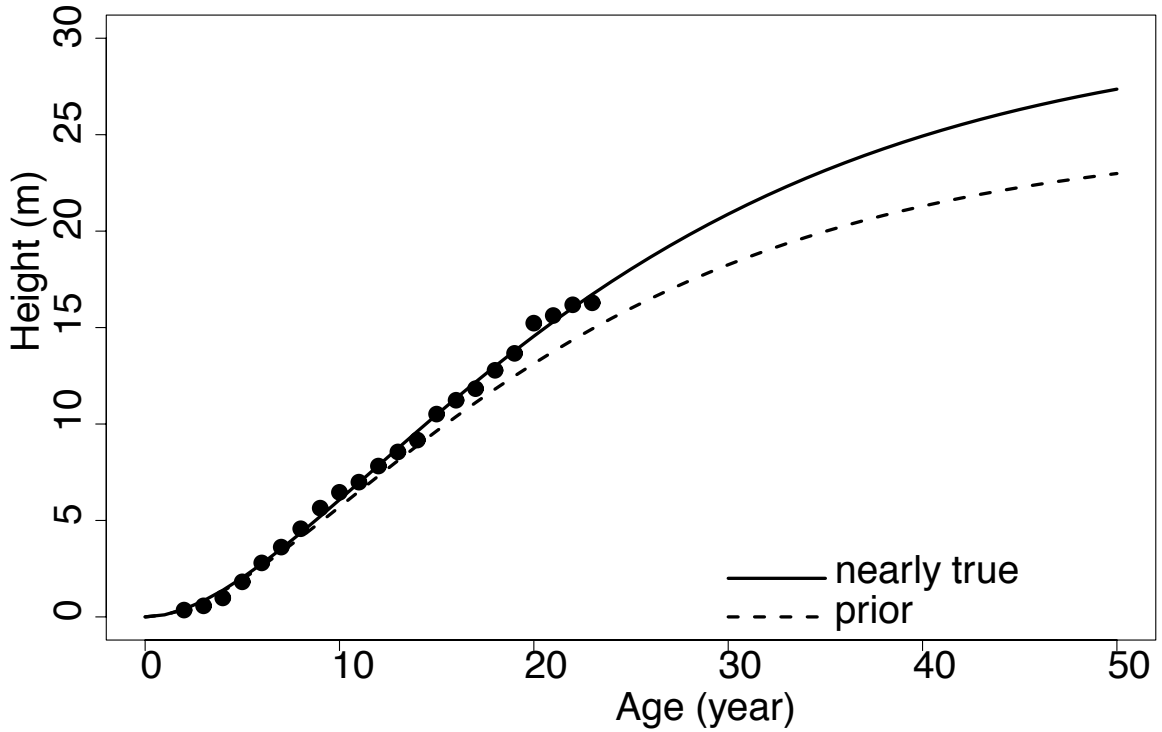


Heights from a study plot in Kyushu



Fitted curve from a lot of measurements (1)

Can we estimate parameter if there are less data?



Bayesean method

- Bayes theorem:

$$f_{Z|X}(z|x) = \frac{f_{X|Z}(x|z) f_Z(z)}{\int f_{X|Z}(x|z) f_Z(z) \mathrm{d}z}$$

  - $f_{Z|X}$ and $f_{X|Z}$: conditional probability (density)
  - f_Z: marginal probability (density)
- By regarding $f_Z(z)$ as prior distribution for $Z = (a, b, c)'$, we get posterior distribution $f_{Z|X}(z|x)$ by Bayes theorem
  - The mode or the mean of $f_{Z|X}(z|x)$ is usually used as a Bayes estimator of Z

Prior distribution for $Z = (a, b, c)'$

- We need to construct a prior distribution $f_Z(z)$
  - Method 1: All parameters in $f_Z(z)$ are estimated by data from other trees
  - Method 2: Values from a density management diagram are used as the mean parameter in $f_Z(z)$, and the other parameters in $f_Z(z)$ are estimated from other trees
- Here we use Method 1 and assume a Gaussian distribution for $f_Z(z)$
- That is, we use $\mathbf{N}(z_0, \Sigma_Z)$ as $f_Z(z)$
  - We estimate z_0 and Σ_Z from other trees
  - Note that we also estimate σ^2 from other trees

Derivation of Bayes estimator

Since $X|Z \sim \mathrm{N}(g(z_0) - A(z_0)(Z - z_0), \Sigma_{X|Z})$ and $Z \sim \mathrm{N}(z_0, \Sigma_Z)$, we have

$$\begin{pmatrix} X \\ Z \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} g(z_0) \\ z_0 \end{pmatrix}, \begin{pmatrix} \Sigma_{X|Z} + A(z_0)\Sigma_Z A(z_0)' & A(z_0)\Sigma_Z \\ \Sigma_Z A(z_0)' & \Sigma_Z \end{pmatrix} \right),$$

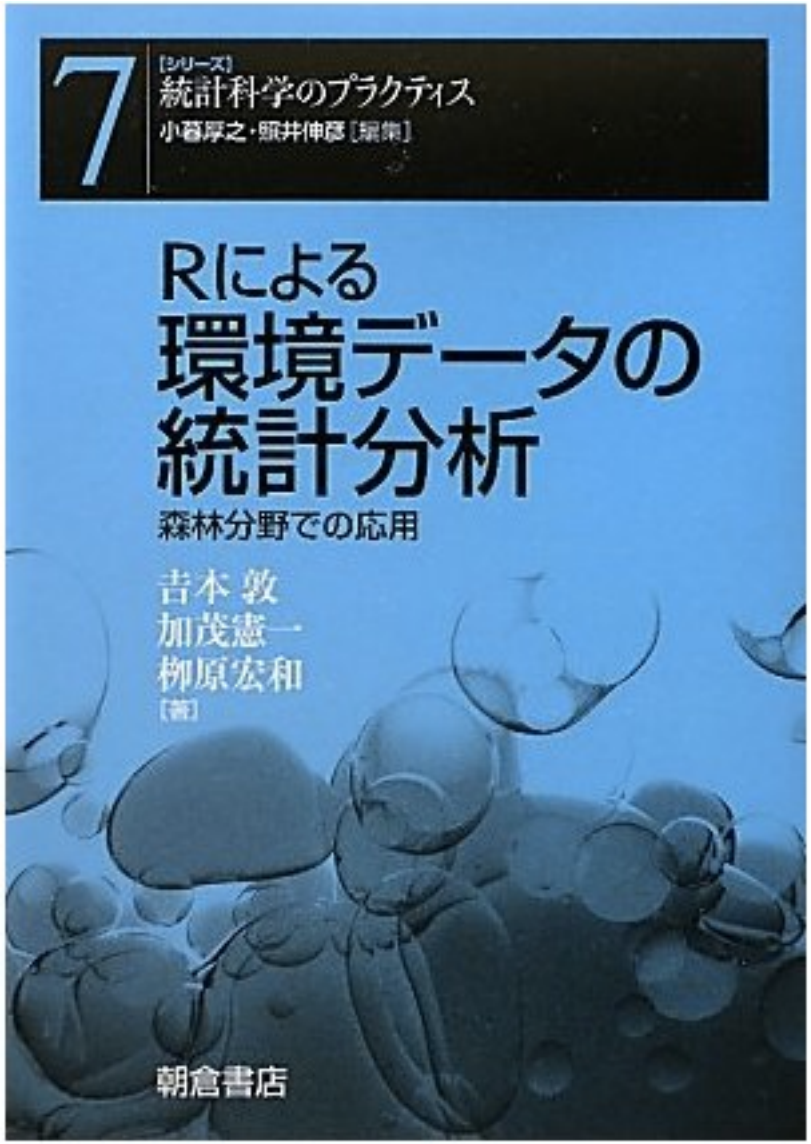
where $A(z_0) = \partial g(z)/\partial z'|_{z=z_0}$, which leads to

$$Z|X \sim \mathrm{N}(\mu_{Z|X}, \Sigma_{Z|X}),$$

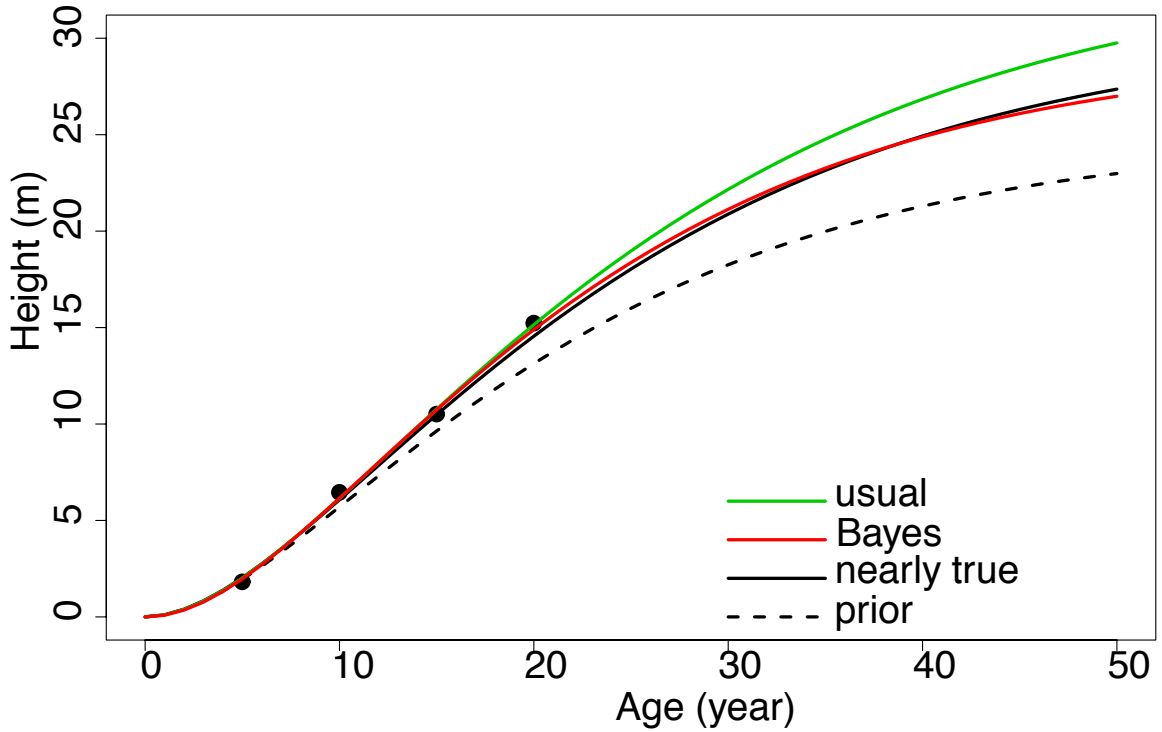
where

$$\begin{aligned}
\Sigma_{Z|X} &= \Sigma_Z - \Sigma_Z A(z_0)'(\Sigma_{X|Z} + A(z_0)\Sigma_Z A(z_0)')^{-1} A(z_0)\Sigma_Z \\
&= (\Sigma_Z^{-1} + A(z_0)'\Sigma_{X|Z}^{-1} A(z_0))^{-1} \\
\mu_{Z|X} &= z_0 + \Sigma_Z A(z_0)'(\Sigma_{X|Z} + A(z_0)\Sigma_Z A(z_0)')^{-1}(X - g(z_0)) \\
&= z_0 + \Sigma_Z A(z_0)'(\Sigma_{X|Z}^{-1} - \Sigma_{X|Z}^{-1} A(z_0)\Sigma_{Z|X} A(z_0)'\Sigma_{X|Z}^{-1})(X - g(z_0)) \\
&= z_0 + \Sigma_{Z|X} A(z_0)'\Sigma_{X|Z}^{-1}(X - g(z_0))
\end{aligned}$$

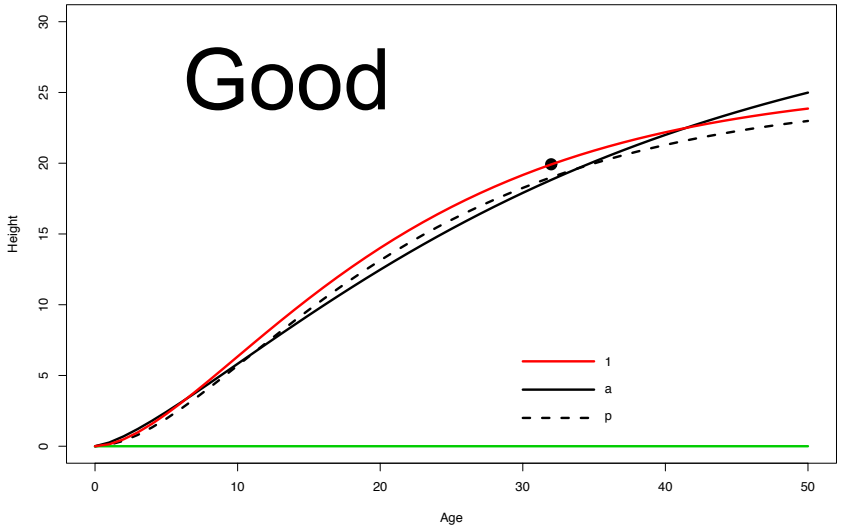
Published in Feb, 2012

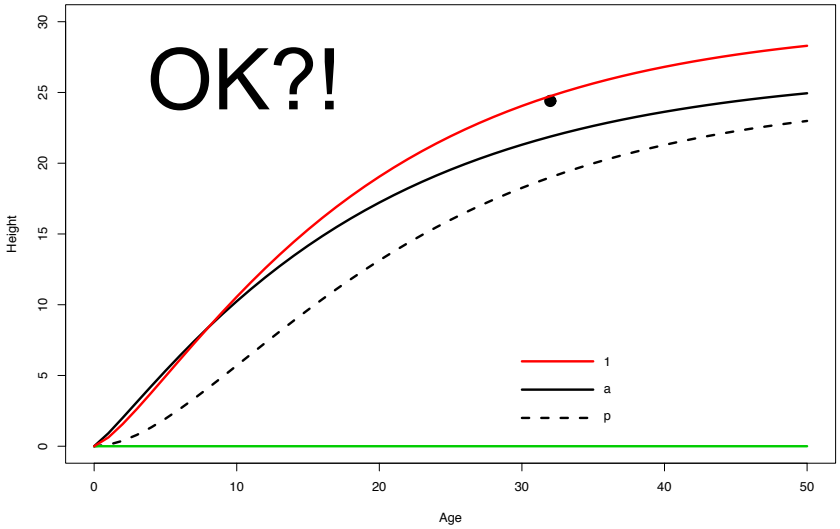


Estimated curve from a few measurements (1)



How about with only one data?



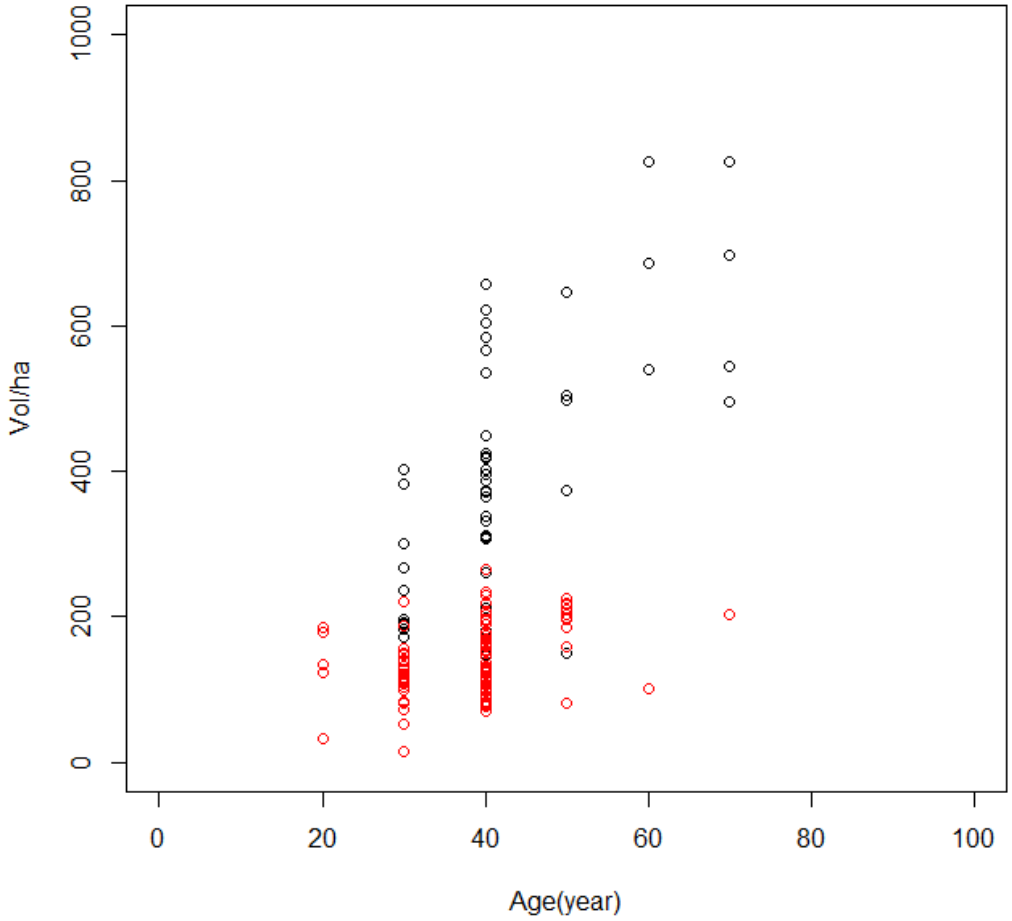


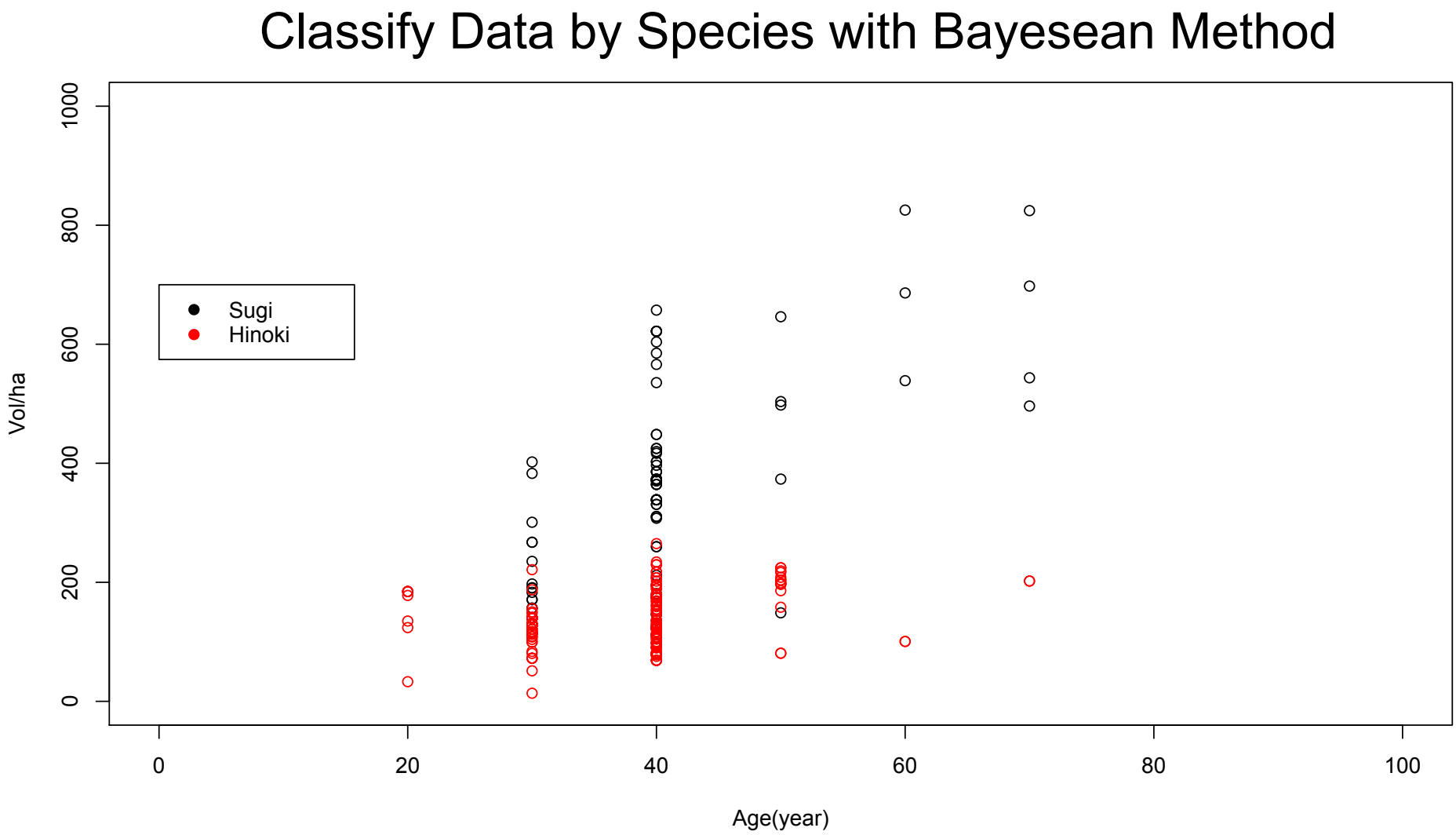
Recursive Approach to Update Prior Information

Volume Data

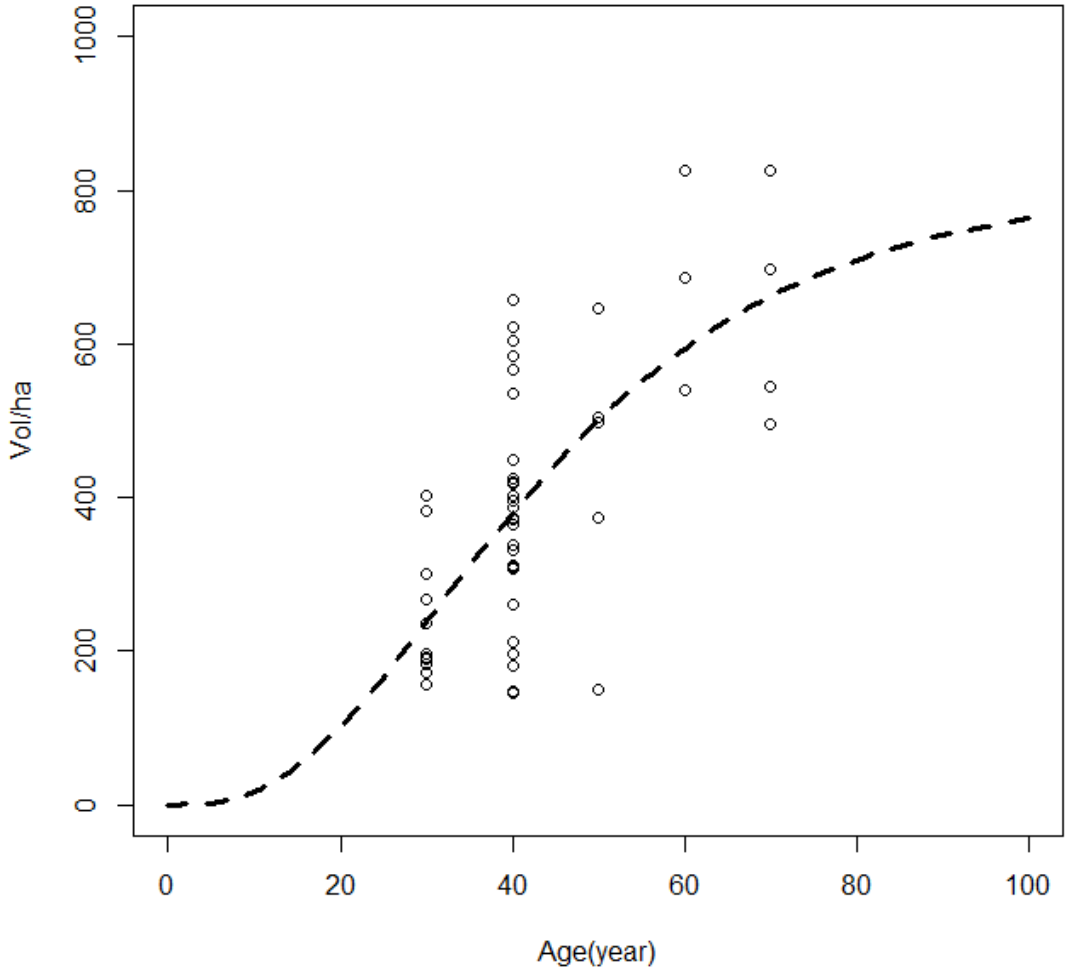
1. Species
2. ID
3. Age of Stands
4. Location
5. Owner
6. Area
7. Volume

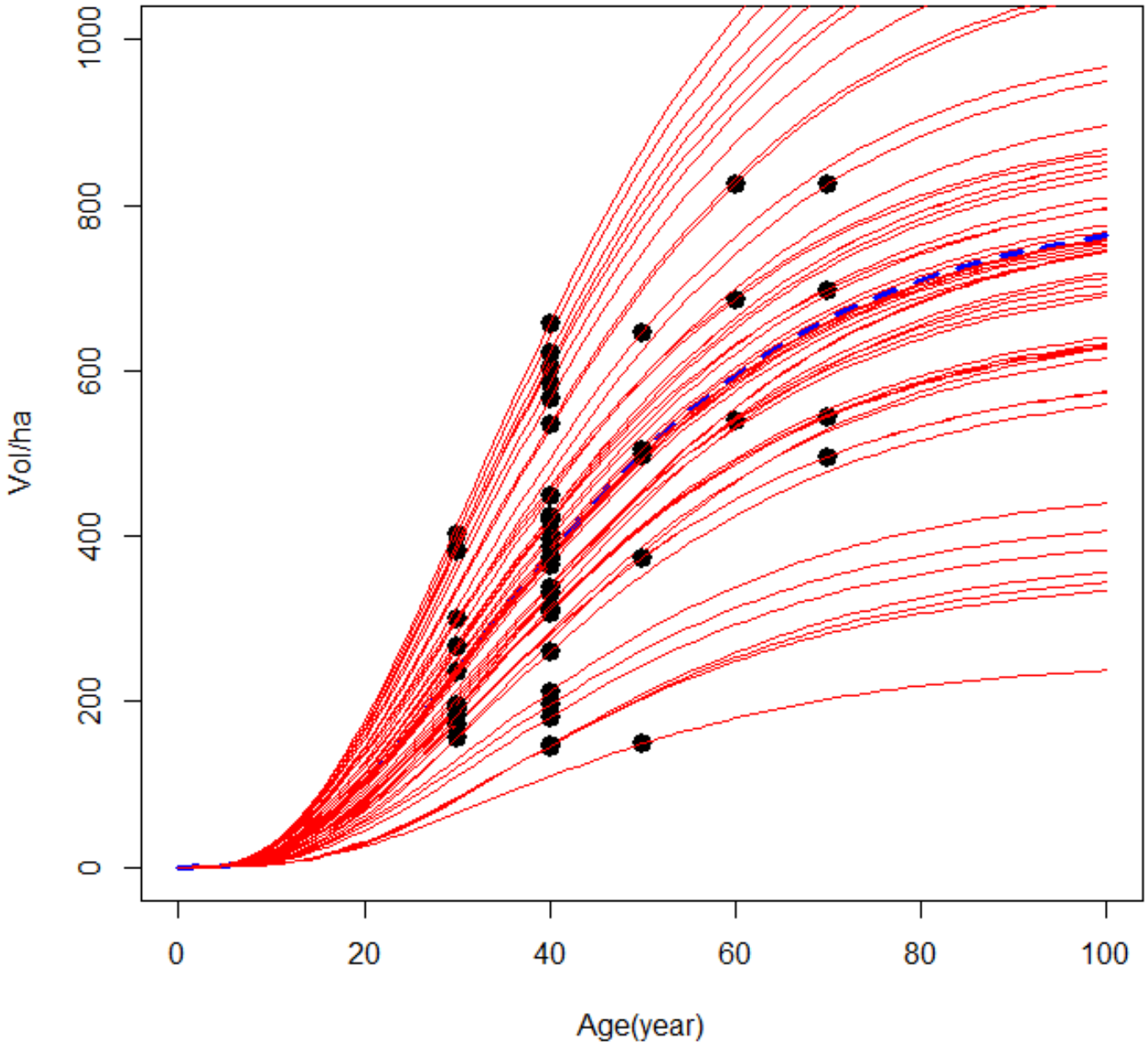
$$V = \frac{Volume}{Area}(\mathrm{m}^3/\mathrm{ha})$$

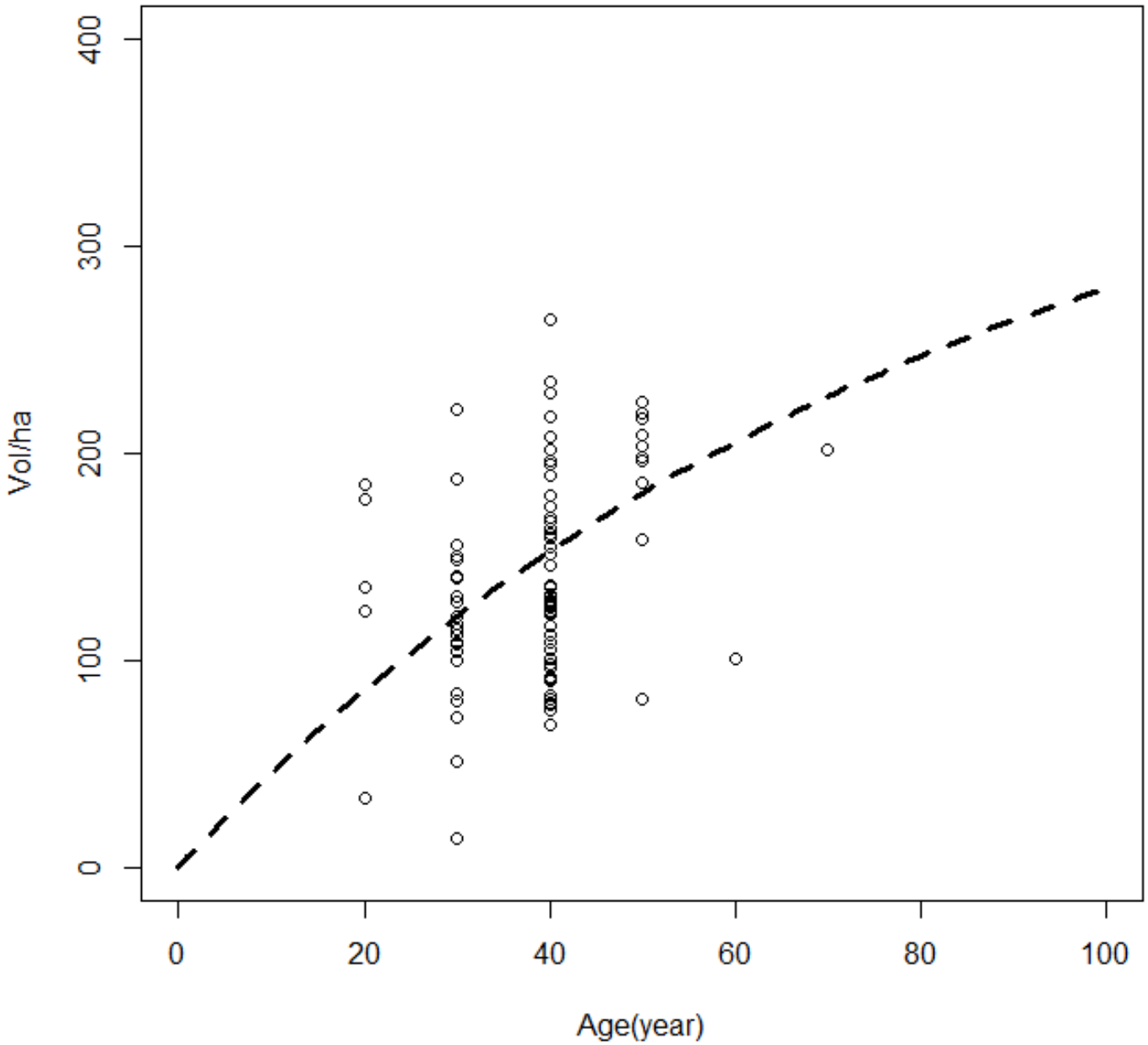



Classify Data by Species with Bayesean Method
Vol/ha
0
200
400
600
800
1000
Sugi
Hinoki
0
20
40
60
80
100
Age(year)

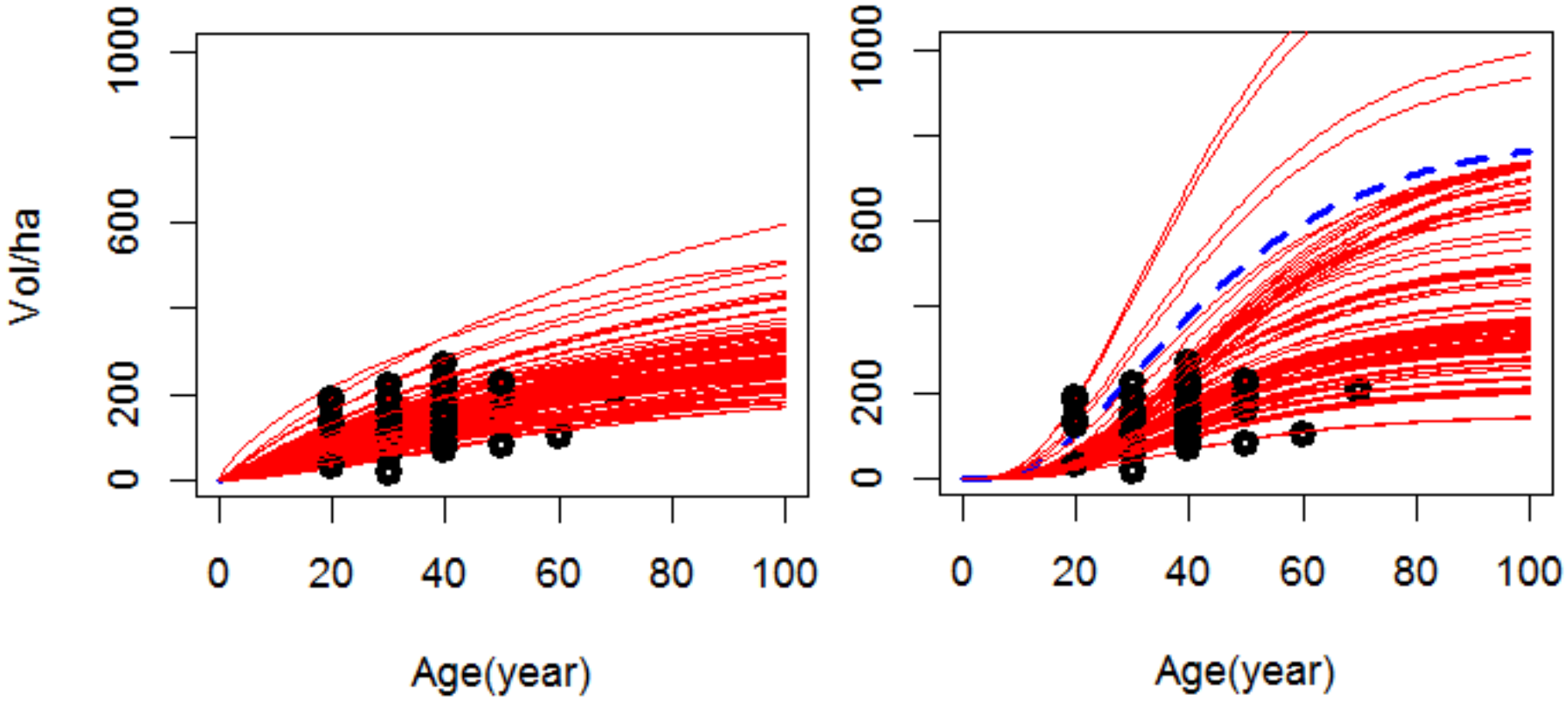
Estimating Growth Curve as Prior Information

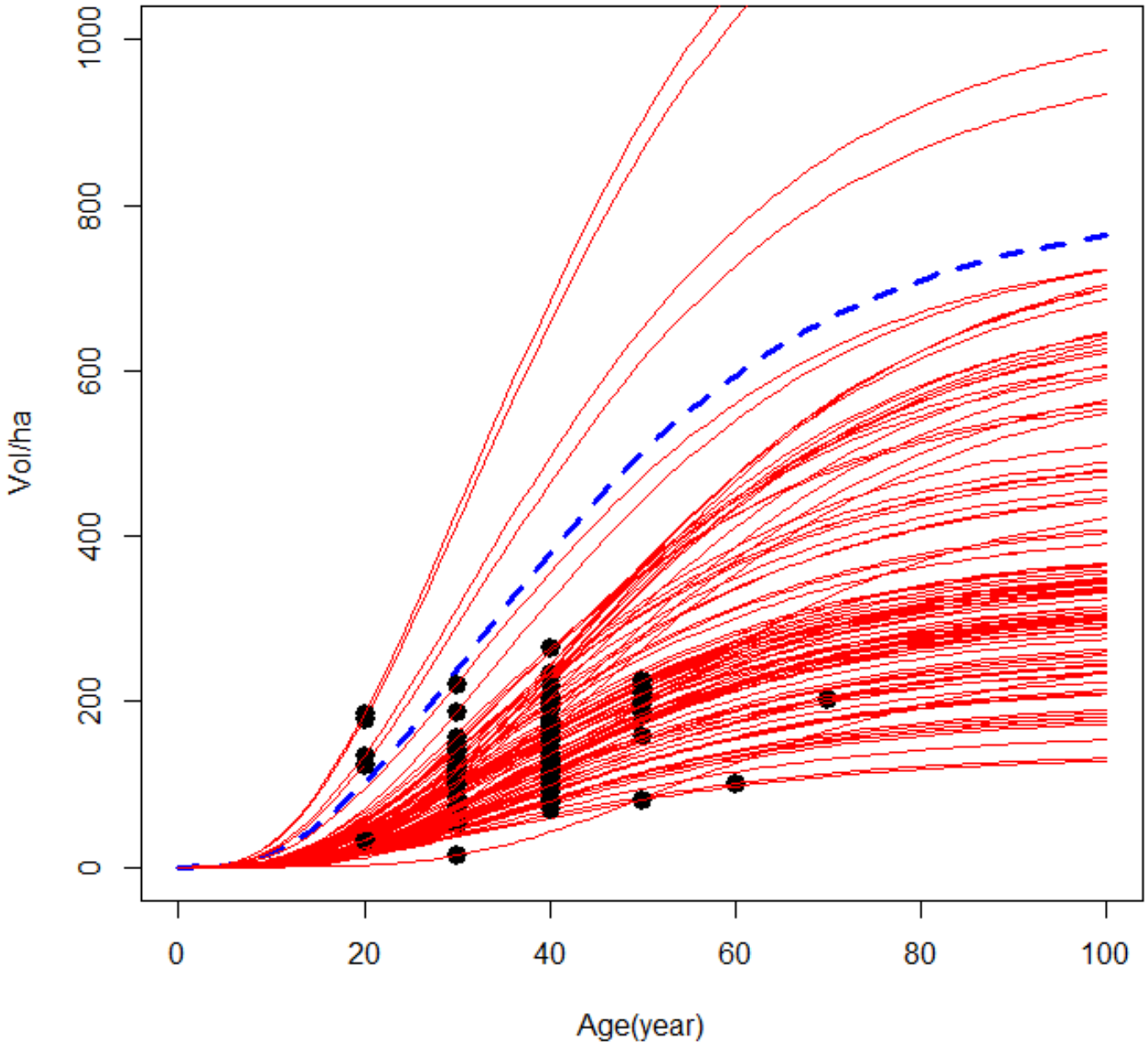



Vol/ha
0
200
400
600
800
1000
0
20
40
60
80
100
Age(year)


Vol/ha
0
100
200
300
400
0
20
40
60
80
100
Age(year)

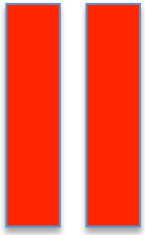
Which is more likely?




Vol/ha
0
200
400
600
800
1000
0
20
40
60
80
100
Age(year)

Growth Projection + Scheduling

Prior Information + Some Data



Acceptable Solutions