森林レベルの最適化

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数理計画法による最適化モデル

線形計画法LP
Strategic Planning

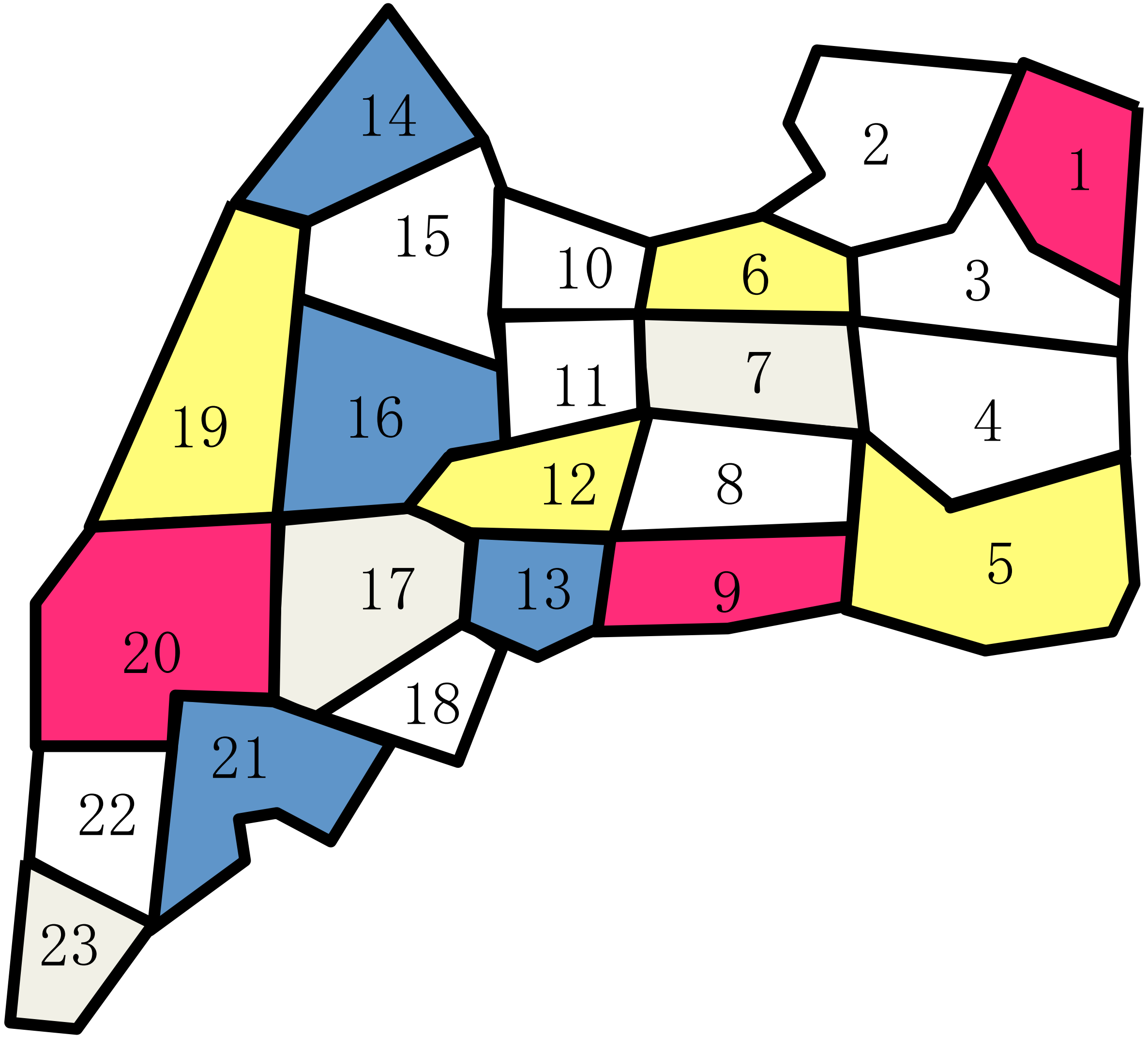
↓

0-1 *整数計画法*
空間制約･場所特定
Tactical Planning

0. オーソドックスな問題：持続性（時間）

複数林分の管理

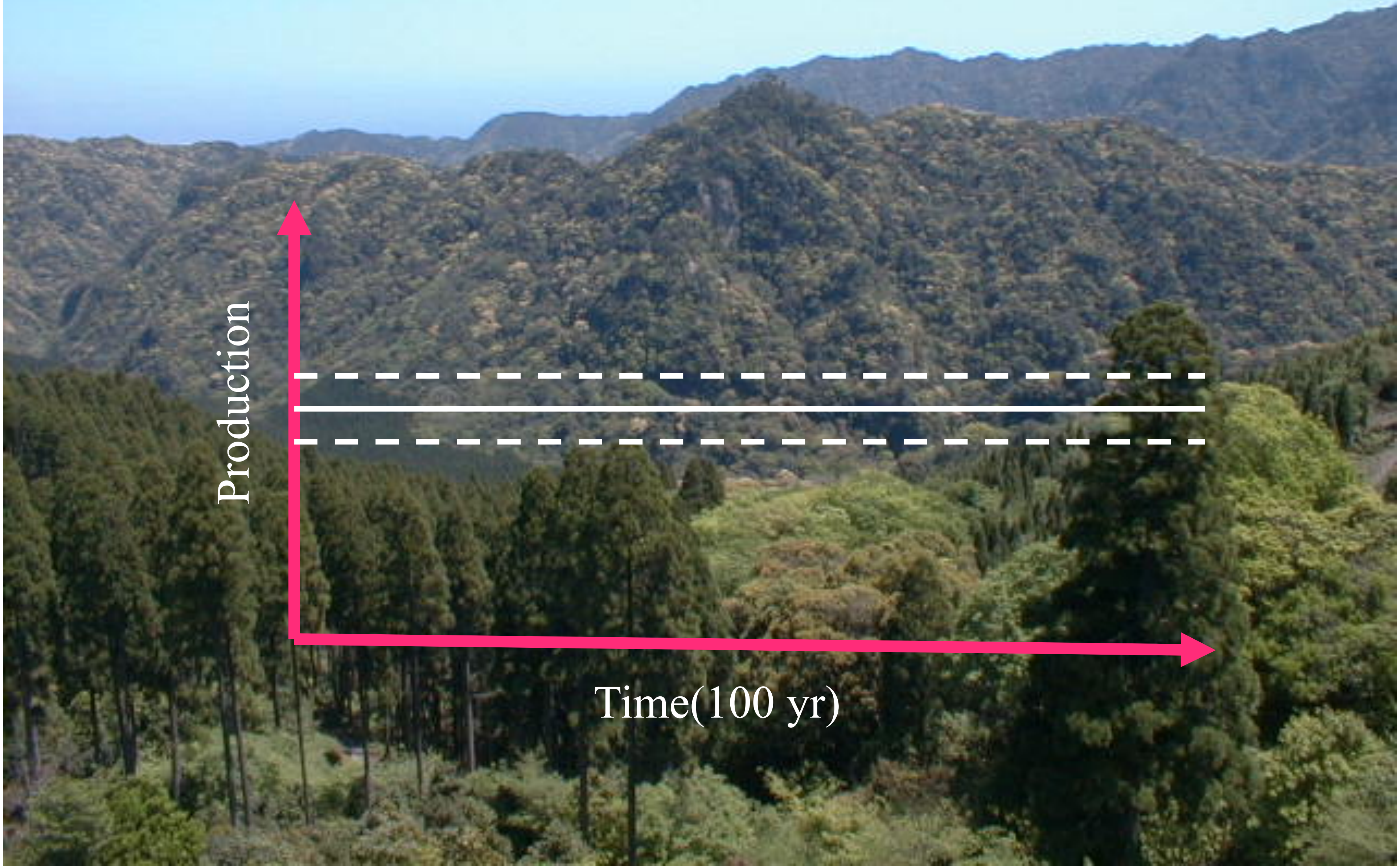
制御時期，場所，強度：時空間を考慮



数理計画法の枠組みで定式化

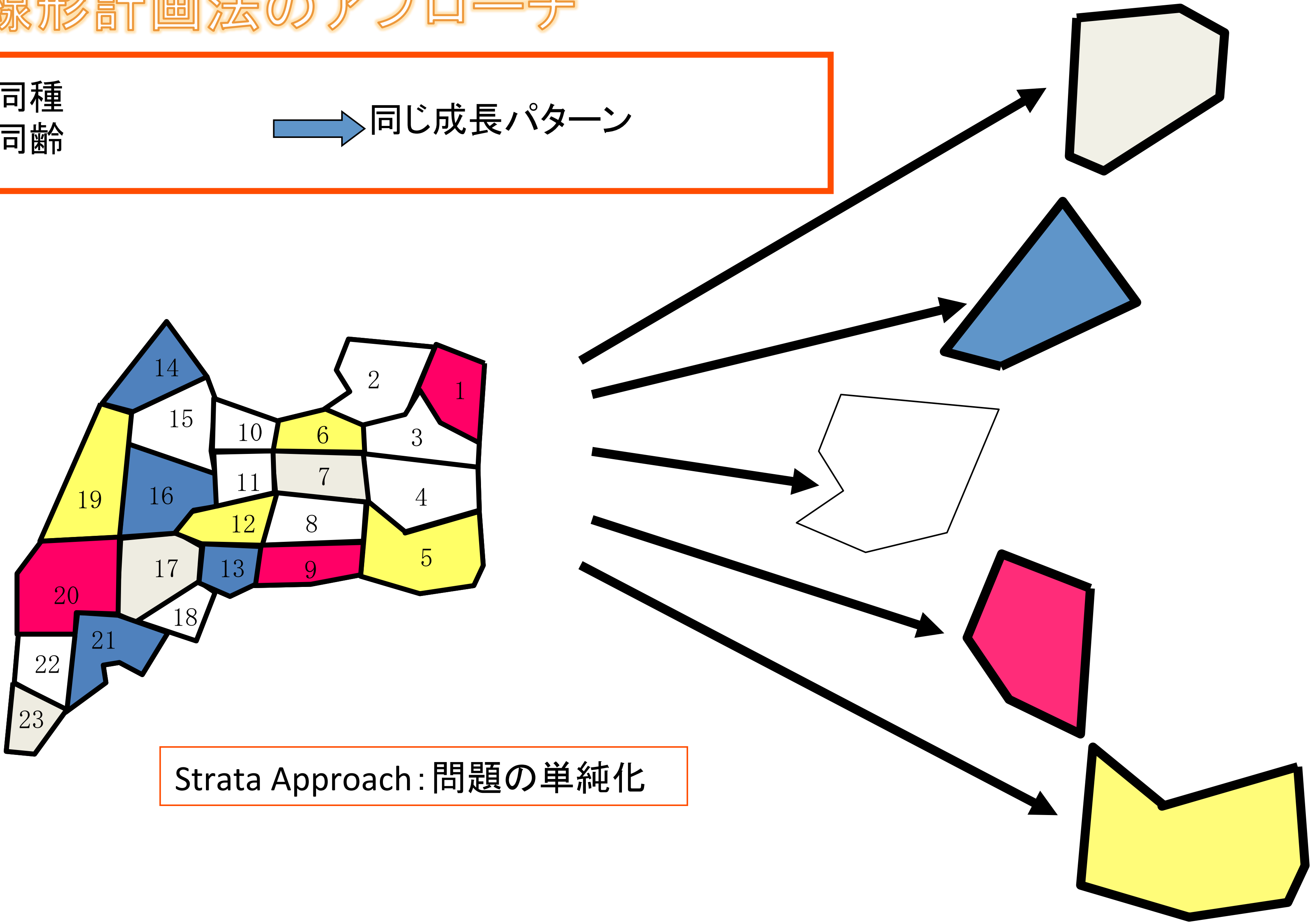
- 決定変数を決める:何を制御するのか？
- 必要な係数を推定する(予測値など)
- 最適化の目的を決める
- 制約条件を列挙する
  - 土地利用に関わる制約
  - 生産量に関わる制約:持続性
  - など

伝統的な持続性の確保
＝＞伐採量の維持



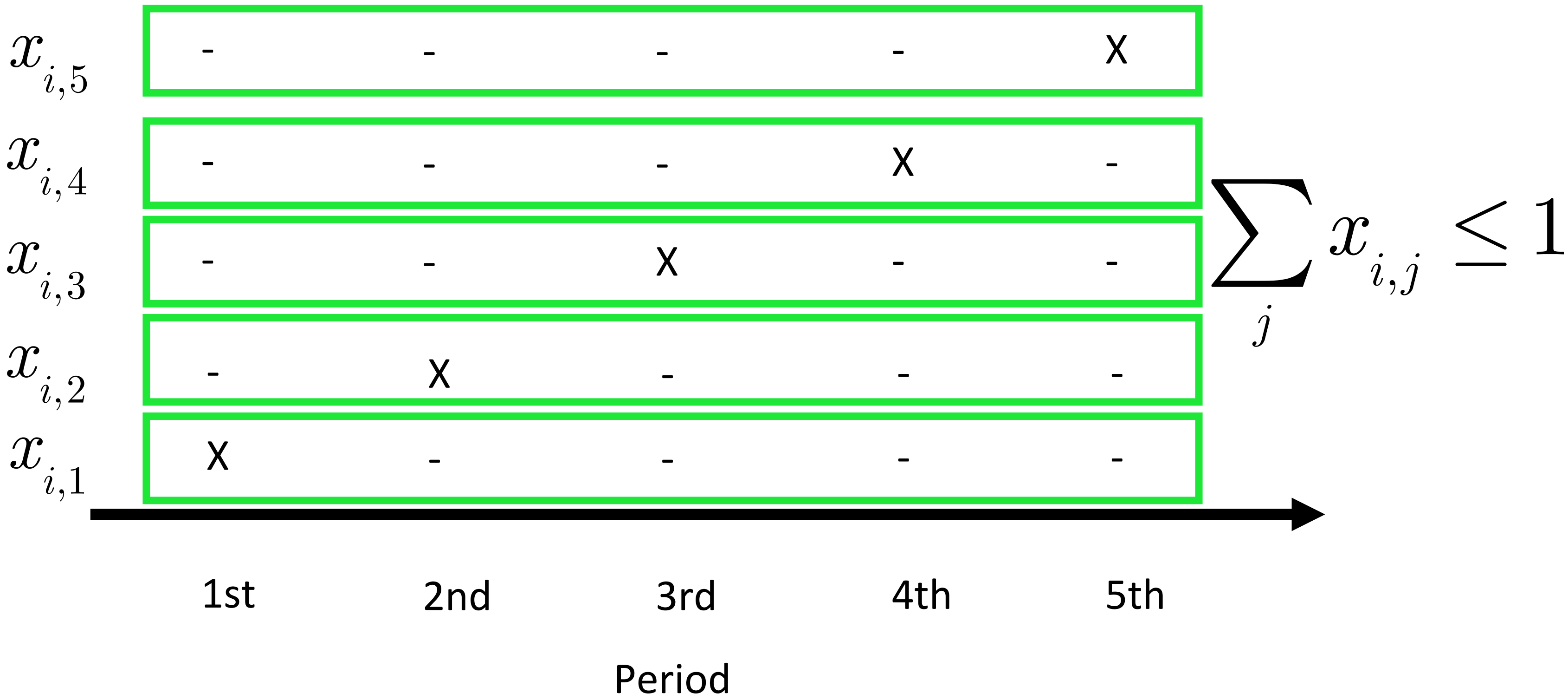
線形計画法のアプローチ

同種
同齢
→同じ成長パターン

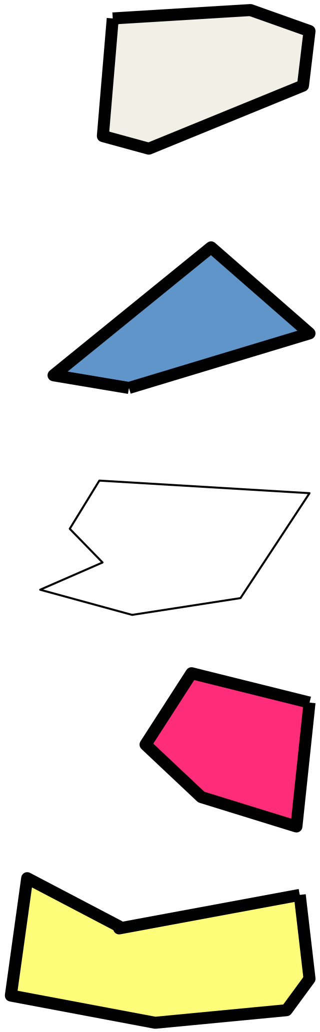


Strata Approach：問題の単純化

決定変数: 施業
各期にどれだけの伐採



各分類でいつ，どれだけ？



	1st	2nd	3rd	4th	5th
	10%	30%	10%	20%	30%
	5%	25%	35%	25%	10%
	30%	0%	30%	30%	10%
	50%	5%	10%	10%	25%
	10%	20%	30%	40%	0%

Time

Definition of variables & coefficients with Single Harvest

Decision variable matrix

$$\boldsymbol{X} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,T} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,T} \end{pmatrix},$$

$x_{i,j}$: portion of the i-th unit to be harvested at period j

Coefficient matrix

$$\boldsymbol{C} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,T} \\ \vdots & \ddots & \vdots \\ c_{m,1} & \cdots & c_{m,T} \end{pmatrix}$$

$c_{i,j}$: coefficient of $x_{i,j}$

Column vectors of all are for the same period

Volume flow matrix

$$\boldsymbol{V} = \begin{pmatrix} v_{1,1} & \cdots & v_{1,T} \\ \vdots & \ddots & \vdots \\ v_{1,m} & \cdots & v_{m,T} \end{pmatrix}$$

$v_{i,j}$: volume flow from $x_{i.j}$ at period j

period j

Objective Function with Single Harvest

To maximize net present value of the sum of all returns from harvests

$$
\begin{aligned}
Z &= \max_{\mathbf{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) \\
&= \max_{\mathbf{X}} \operatorname{tr}\begin{pmatrix} c_{1,1}x_{1,1} + \cdots + c_{m,1}x_{m,1} & \cdots & \cdots \\ \cdots & \ddots & \cdots \\ \cdots & \cdots & c_{1,T}x_{1,T} + \cdots + c_{m,T}x_{m,T} \end{pmatrix} \\
&= \max_{\mathbf{X}} \sum_{i=1}^{m}\sum_{j=1}^{T} c_{i,j} \cdot x_{i,j}
\end{aligned}
$$

Note: trace of a square matrix is the summation of diagonal elements

$$
A = \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix} \quad \Rightarrow \quad \operatorname{tr}(A) = \sum_{i=1}^{n} a_{i,i}
$$

Land Accounting Constraints with Single Harvest

Sum of portion <=0

$$x_{i,1} + x_{i,2} \cdots x_{i,T-1} + x_{i,T} \leq 1 \qquad i = 1, \ldots, m$$

or

$$\begin{pmatrix} x_{1,1} & \cdots & x_{1,T} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,T} \end{pmatrix} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}_T \leq \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}_m \Leftrightarrow \boxed{\boldsymbol{X}\mathbf{1}_T \leq \mathbf{1}_m}$$

Harvest Flow Constraints with Single Harvest

Even flow over time

$$v_{1,j-1}x_{1,j-1} + \cdots + v_{m,j-1}x_{m,j-1} = v_{1,j}x_{1,j} + \cdots + v_{m,j}x_{m,j}$$

or

$$\begin{pmatrix} v_{1,j-1} & \cdots & v_{m,j-1} \end{pmatrix} \begin{pmatrix} x_{1,j-1} \\ \vdots \\ x_{m,j-1} \end{pmatrix} = \begin{pmatrix} v_{1,j} & \cdots & v_{m,j} \end{pmatrix} \begin{pmatrix} x_{1,j} \\ \vdots \\ x_{m,j} \end{pmatrix}$$

or

$$\boldsymbol{v}'^{(j-1)}\boldsymbol{x}^{(j-1)} = \boldsymbol{v}'^{(j)}\boldsymbol{x}^{(j)}$$

線形計画法による定式化

Max PNV

$$Z = \max_{\mathbf{X}} \mathrm{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m} \sum_{j=1}^{T} c_{i,j} \cdot x_{i,j}$$

$st.$

$$\boldsymbol{X}\mathbf{1}_T \le \mathbf{1}_m$$ 土地利用制約

$$\boldsymbol{v}'^{(p-1)}\boldsymbol{x}^{(p-1)} = \boldsymbol{v}'^{(p)}\boldsymbol{x}^{(p)} \quad , \quad p = 2, \cdots, T$$

where 生産量制約

$$\boldsymbol{v}^{(p)} = (v_{1,p}, v_{2,p} \cdots, v_{m,p})', \ \boldsymbol{x}^{(p)} = (x_{1,p}, x_{2,p} \cdots, x_{m,p})'$$

$\boldsymbol{v}^{(p)}$ and $\boldsymbol{x}^{(p)}$ are the p-th column vector of $\boldsymbol{V}$ and $\boldsymbol{X}$, respectively

最適化ソフトウェア使用
Cplex, Gurobi, SCIP
(Academic Free)

Note: trace of a square matrix is the summation of diagonal elements

$$A = \begin{pmatrix} a_{1,1} & \cdots & a_{1,n} \\ \vdots & \ddots & \vdots \\ a_{n,1} & \cdots & a_{n,n} \end{pmatrix} \quad \Rightarrow \quad \mathrm{tr}(A) = \sum_{i=1}^{n} a_{i,i}$$

複数回の伐採を考慮: Model I

Table 1: Example of treatments

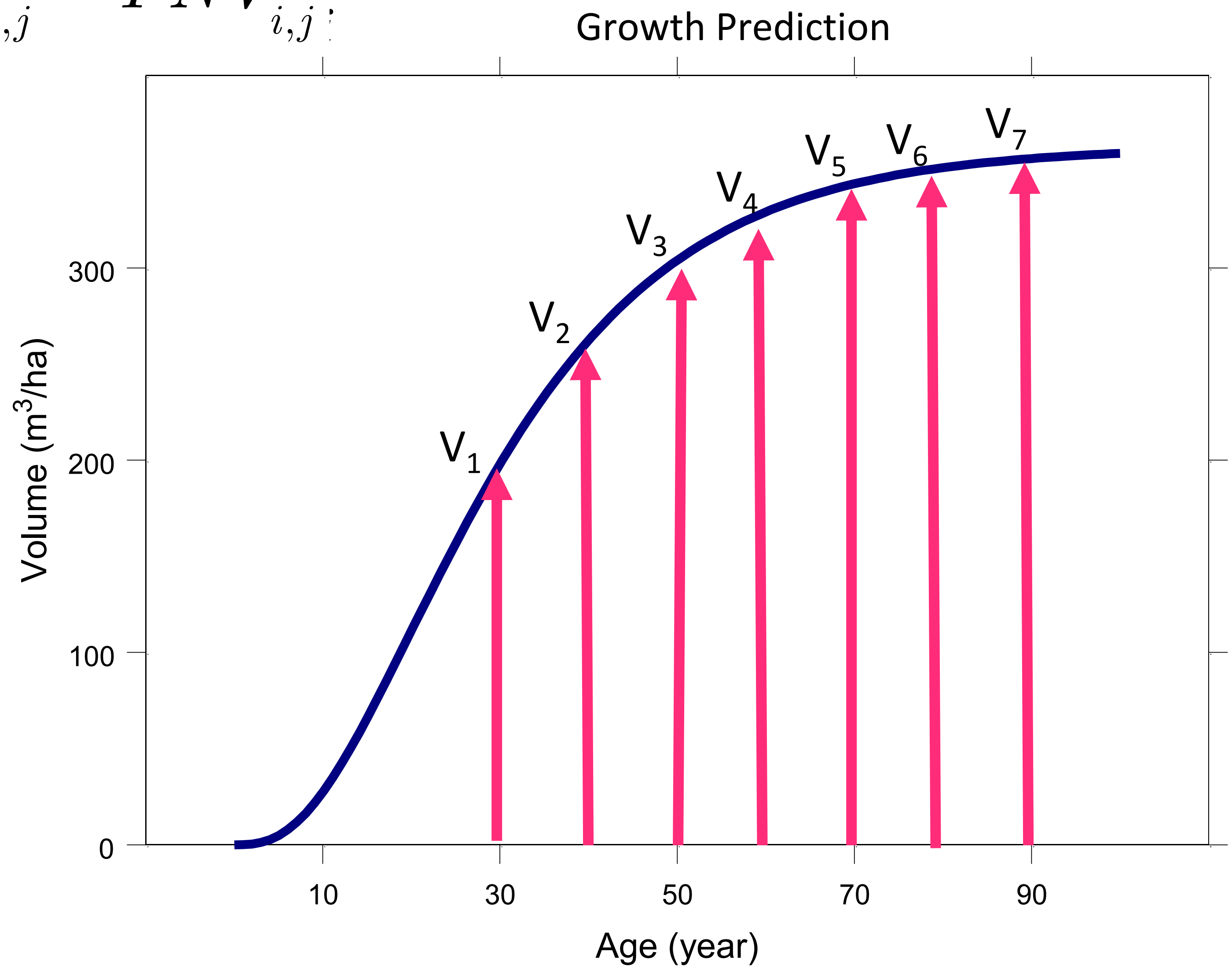
Minimum Rotation: 6 periods over 10 periods

	Treatment No.	Decision Variable	Coefficient	Period									
				1	2	3	4	5	6	7	8	9	10
Treatment	1	$x_{i,1}$	$c_{i,1}$	X	0	0	0	0	0	0	0	0	0
	2	$x_{i,2}$	$c_{i,2}$	X	0	0	0	0	0	X	0	0	0
	3	$x_{i,3}$	$c_{i,3}$	X	0	0	0	0	0	0	X	0	0
	4	$x_{i,4}$	$c_{i,4}$	X	0	0	0	0	0	0	0	X	0
	5	$x_{i,5}$	$c_{i,5}$	X	0	0	0	0	0	0	0	0	X
	6	$x_{i,6}$	$c_{i,6}$	0	X	0	0	0	0	0	0	0	0
	7	$x_{i,7}$	$c_{i,7}$	0	X	0	0	0	0	0	X	0	0
	8	$x_{i,8}$	$c_{i,8}$	0	X	0	0	0	0	0	0	X	0
	9	$x_{i,9}$	$c_{i,9}$	0	X	0	0	0	0	0	0	0	X
	10	$x_{i,10}$	$c_{i,10}$	0	0	X	0	0	0	0	0	0	0
	11	$x_{i,11}$	$c_{i,11}$	0	0	X	0	0	0	0	0	X	0
	12	$x_{i,12}$	$c_{i,12}$	0	0	X	0	0	0	0	0	0	X
	13	$x_{i,13}$	$c_{i,13}$	0	0	0	X	0	0	0	0	0	0
	14	$x_{i,14}$	$c_{i,14}$	0	0	0	X	0	0	0	0	0	X
	15	$x_{i,15}$	$c_{i,15}$	0	0	0	0	X	0	0	0	0	0
	16	$x_{i,16}$	$c_{i,16}$	0	0	0	0	0	X	0	0	0	0
	17	$x_{i,17}$	$c_{i,17}$	0	0	0	0	0	0	X	0	0	0
	18	$x_{i,18}$	$c_{i,18}$	0	0	0	0	0	0	0	X	0	0
	19	$x_{i,19}$	$c_{i,19}$	0	0	0	0	0	0	0	0	X	0
	20	$x_{i,20}$	$c_{i,20}$	0	0	0	0	0	0	0	0	0	X

Note: X denotes harvesting while 0 denotes no harvesting

Estimating Coefficients Value c

$$c_{i,j} = PNV_{i,j}$$



The i-th Coefficient: $c_{i,j}$

	PNV	1	2	3	4	5	6	7	8	9	10
$c_{i,j} PNV_{i,j}$		$P \bullet V_1$	$P \bullet V_2$	$P \bullet V_3$	$P \bullet V_4$	$P \bullet V_5$	$P \bullet V_6$	$P \bullet V_7$	$P \bullet V_8$	$P \bullet V_9$	$P \bullet V_{10}$

$$PNV_{i,j} = \sum_{t=1}^{10} \frac{P \cdot V_{i,t} \cdot A_i}{(1+r)^{d \cdot (t-1)}}$$

PNV from the i-th variable $= PNV_{i,j} \cdot x_{i,j}$

Volume at each period

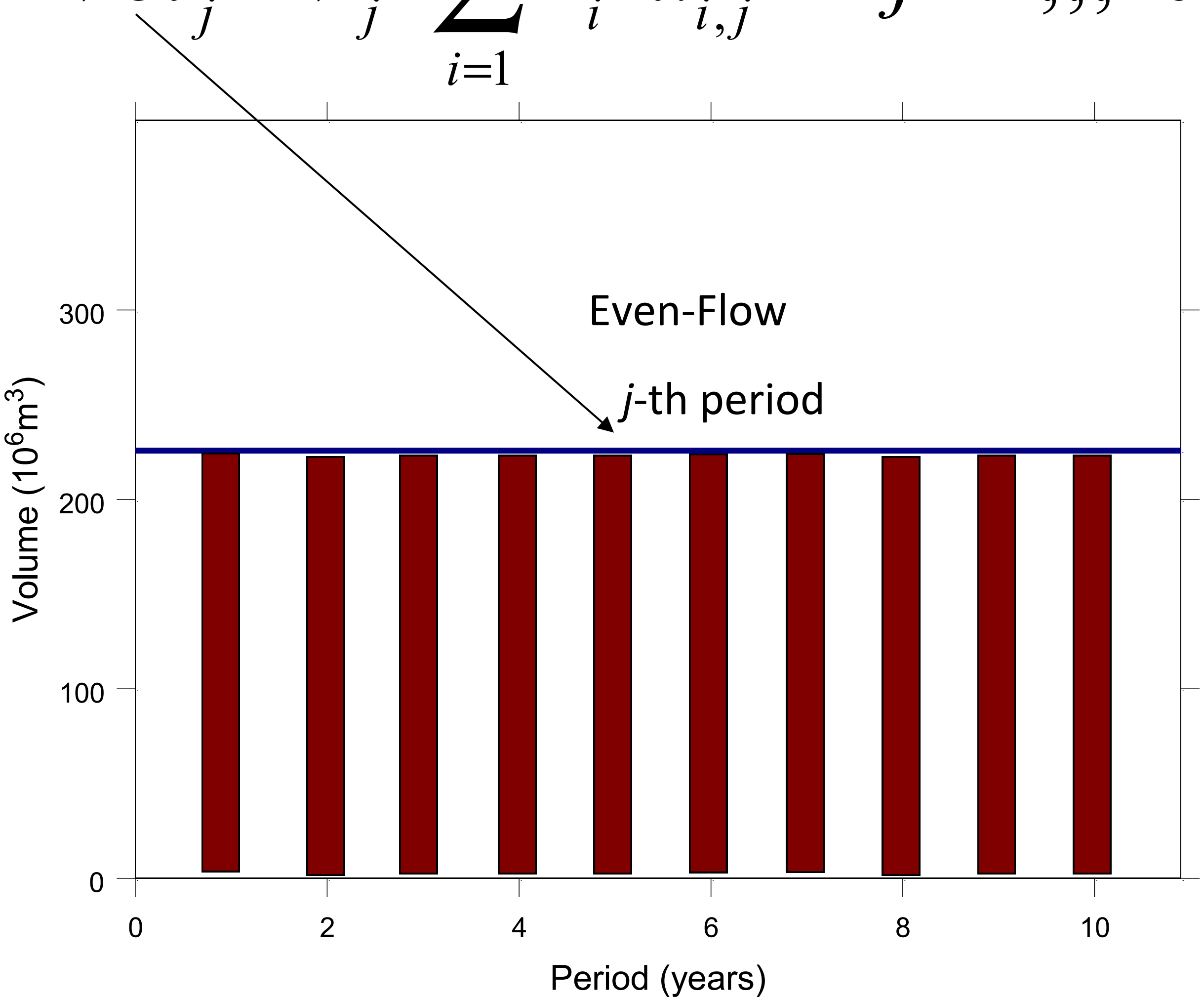
Periods

Treatment

	1	2	3	4	5	6	7	8	9	10
1	V_1	0	0	0	0	0	0	0	0	0
2	V_1	0	0	0	0	0	V_1	0	0	0
3	V_1	0	0	0	0	0	0	V_2	0	0
4	V_1	0	0	0	0	0	0	0	V_3	0
5	V_1	0	0	0	0	0	0	0	0	V_4
6	0	V_2	0	0	0	0	0	0	0	0
7	0	V_2	0	0	0	0	0	V_1	0	0
8	0	V_2	0	0	0	0	0	0	V_2	0
9	0	V_2	0	0	0	0	0	0	0	V_3
10	0	0	V_3	0	0	0	0	0	0	0
11	0	0	V_3	0	0	0	0	0	V_1	0
12	0	0	V_3	0	0	0	0	0	0	V_2
13	0	0	0	V_4	0	0	0	0	0	0
14	0	0	0	V_4	0	0	0	0	0	6
15	0	0	0	0	V_5	0	0	0	0	0
16	0	0	0	0	0	V_6	0	0	0	0
17	0	0	0	0	0	0	V_7	0	0	0
18	0	0	0	0	0	0	0	V_8	0	0
19	0	0	0	0	0	0	0	0	V_9	0
20	0	0	0	0	0	0	0	0	0	V_{10}
Vol	Vol_1	Vol_2	Vol_3	Vol_4	Vol_5	Vol_6	Vol_7	Vol_8	Vol_9	Vol_{10}

Volume flow

$$Vol_j = V_j \cdot \sum_{i=1}^{10} A_i \cdot x_{i,j} \qquad j = 1,,,10$$



Introduction of Treatment

Decision variable matrix

$$\boldsymbol{X} = \begin{pmatrix} x_{1,1} & \cdots & x_{1,n} \\ \vdots & \ddots & \vdots \\ x_{m,1} & \cdots & x_{m,n} \end{pmatrix}$$

$x_{i,j}$: portion of the j-th treatment is implemented for the i-th unit

Coefficient matrix

$$\boldsymbol{C} = \begin{pmatrix} c_{1,1} & \cdots & c_{1,n} \\ \vdots & \ddots & \vdots \\ c_{m,1} & \cdots & c_{m,n} \end{pmatrix}$$

$c_{i,j}$: coefficient of $x_{i,j}$

Volume flow matrix at period p

$$\boldsymbol{V}_p = \begin{pmatrix} v^{(p)}_{1,1} & \cdots & v^{(p)}_{1,n} \\ \vdots & \ddots & \vdots \\ v^{(p)}_{1,m} & \cdots & v^{(p)}_{m,n} \end{pmatrix}$$

$v^{(p)}_{i,j}$: volume flow from $x_{i.j}$

Model I: Linear Programming Formulation

Max PNV

$$Z = \max_{\mathbf{X}} \operatorname{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{i,j} \cdot x_{i,j}$$

$$st.$$

$$\boldsymbol{X}\mathbf{1}_{n} \leq \mathbf{1}_{m}$$

Land Accounting Constraints

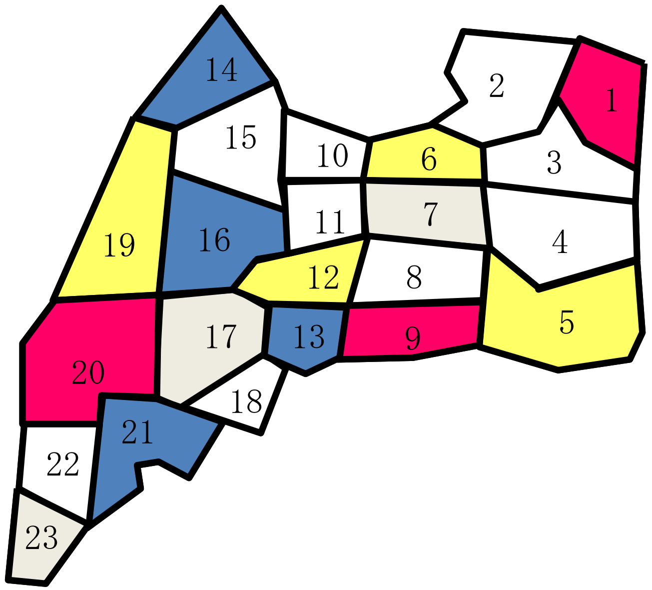
$$\operatorname{tr}(\boldsymbol{V}_{p}'\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{n} v_{i,j}^{(p)} \cdot x_{i,j} = v_{0}, p = 1, \cdots, T$$

Harvest Even-Flow Constraints

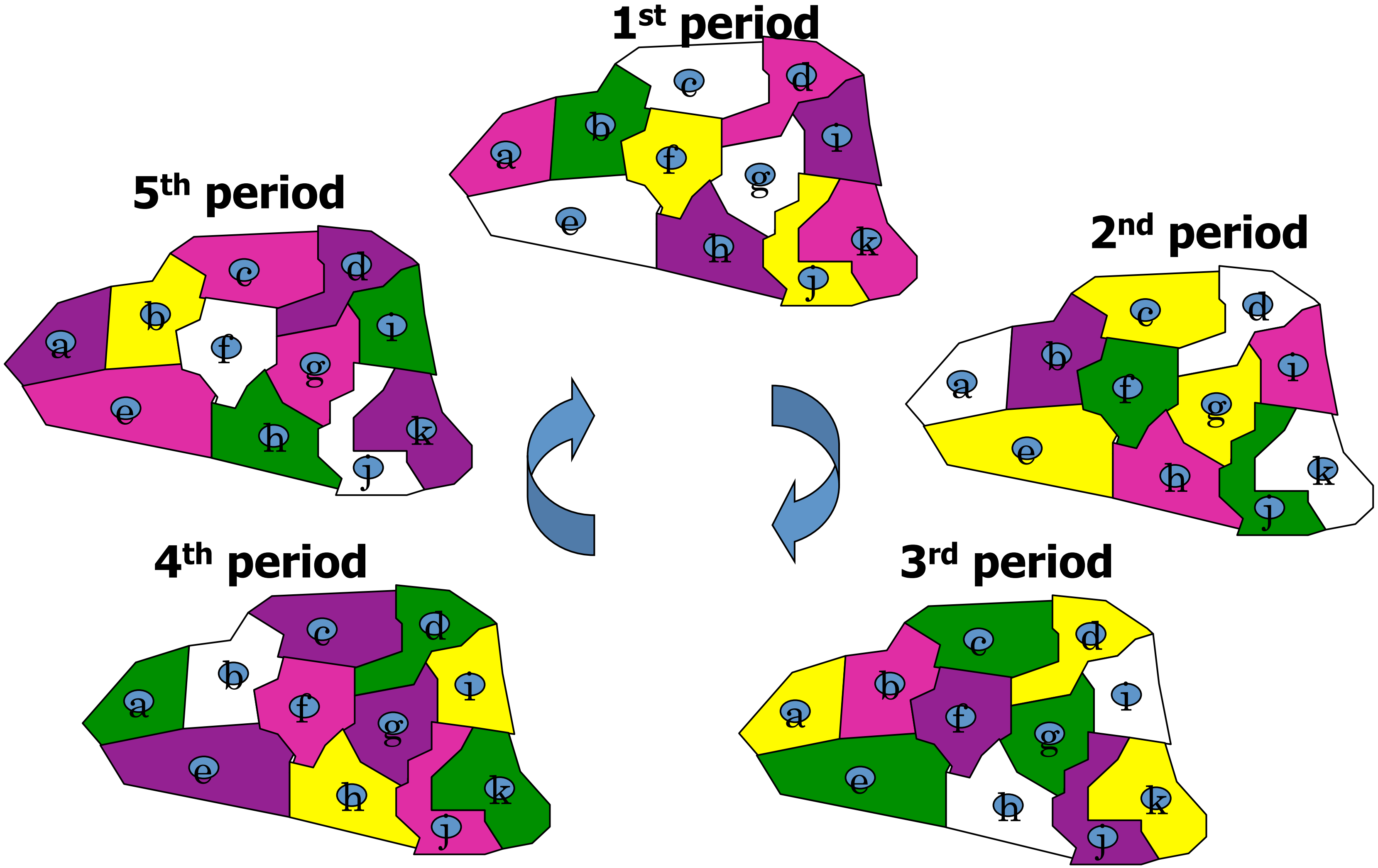
空間的配置：Tactical Planning

いつ、どれだけ、どこで?
位置を考慮する必要がある

同樹種 同齢 同林分	→	同じ成長 伐採 – 伐採なし (0-1)



場所特定のスケジューリング



時空間的配置

0-1 整数計画法による定式化

Decision Variable

Continuous => Discrete

時空間的配置

0-1 整数計画法による定式化

Max PNV

$$Z = \max_{\mathbf{X}} \mathrm{tr}(\boldsymbol{C}'\boldsymbol{X}) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{i,j} \cdot x_{i,j}$$

$$st.$$

$$\boldsymbol{X}\mathbf{1}_n \le \mathbf{1}_m$$ Land Accounting Constraints

$$(1-\alpha)v_0 \le \mathrm{tr}(\boldsymbol{V}_p'\boldsymbol{X}) \le (1+\alpha)v_0, \quad p = 1, \cdots, T$$

Harvest Flow Constraints

$$x_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for the } i\text{-th stand} \\ 0 & \text{otherwise} \end{cases}$$