団地化での森林資源管理

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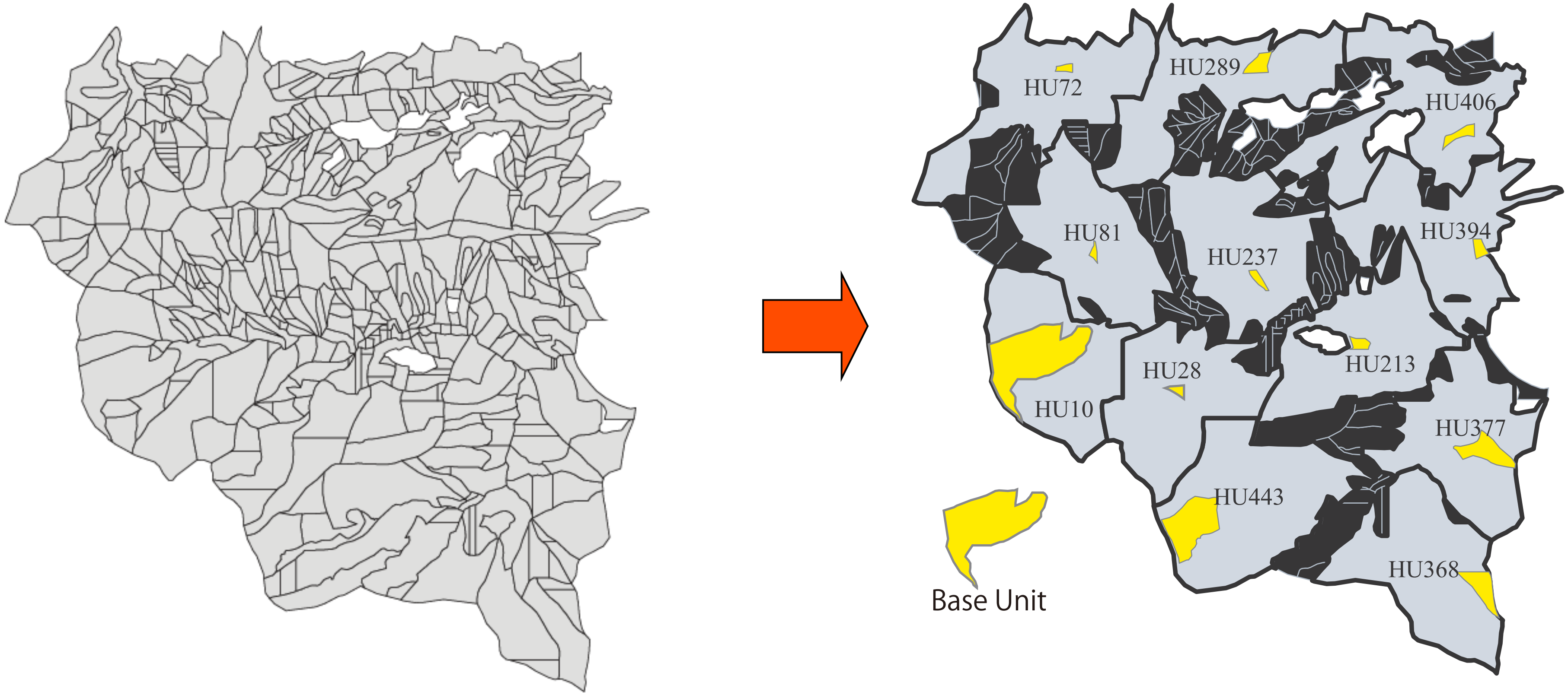
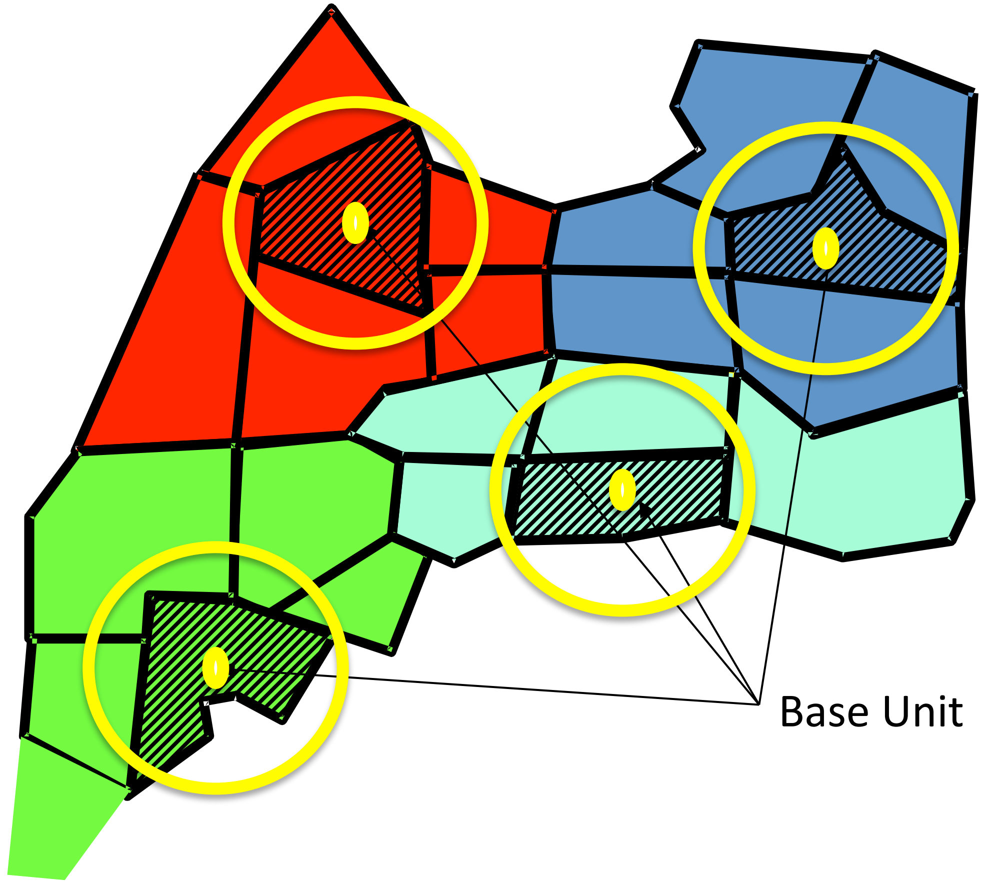
集約化：団地化

どの林分群を集約（団地化）するのか？

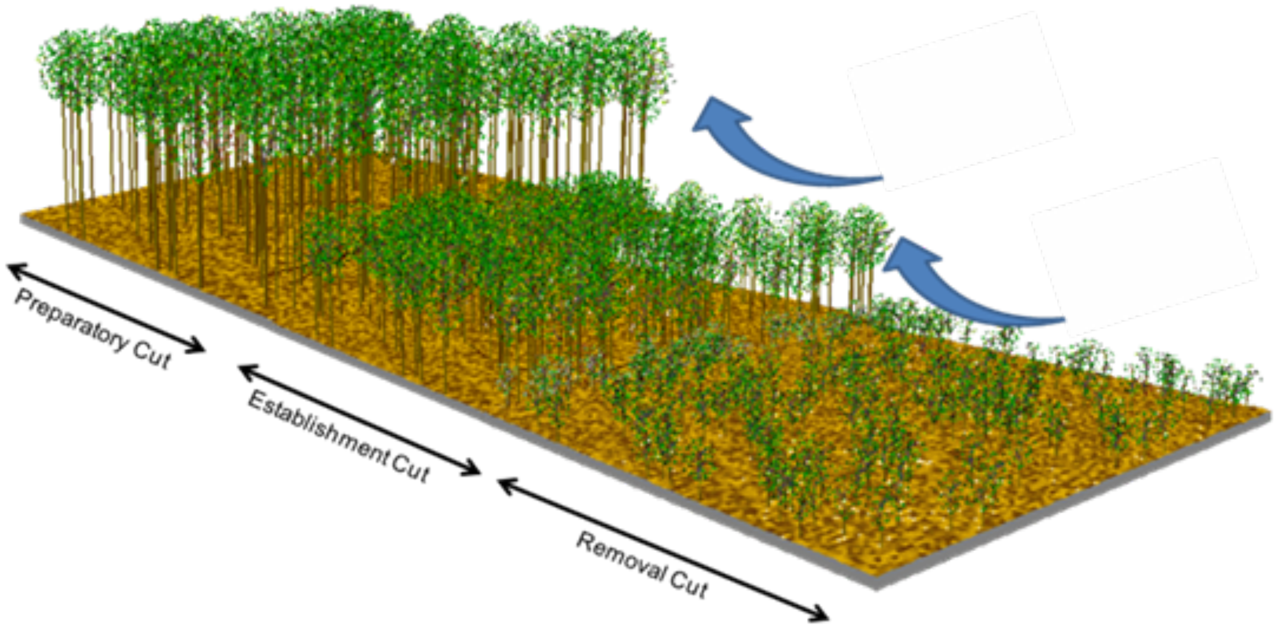


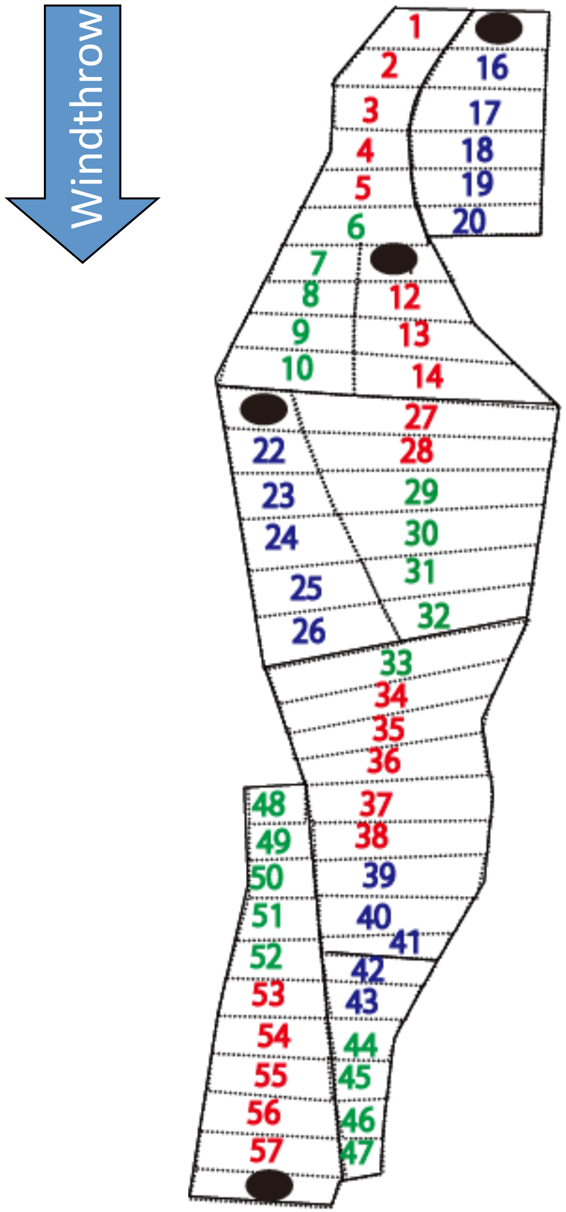
時空間的最適化モデル

集約化された場所での施業時期を考慮
解法：集約候補の生成により対応



傘伐管理
3 cut system

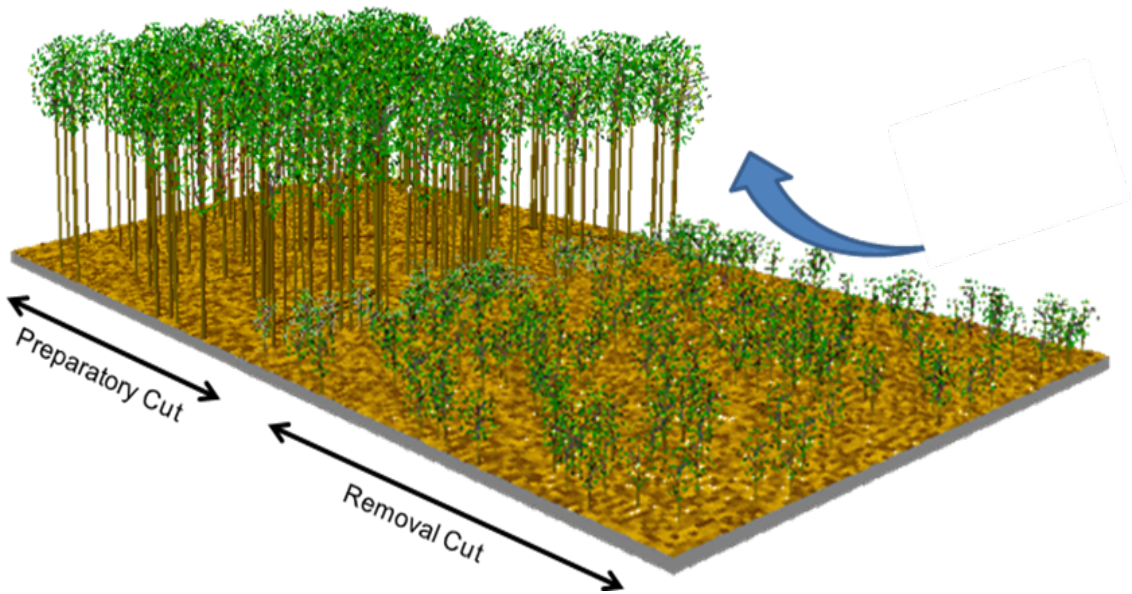




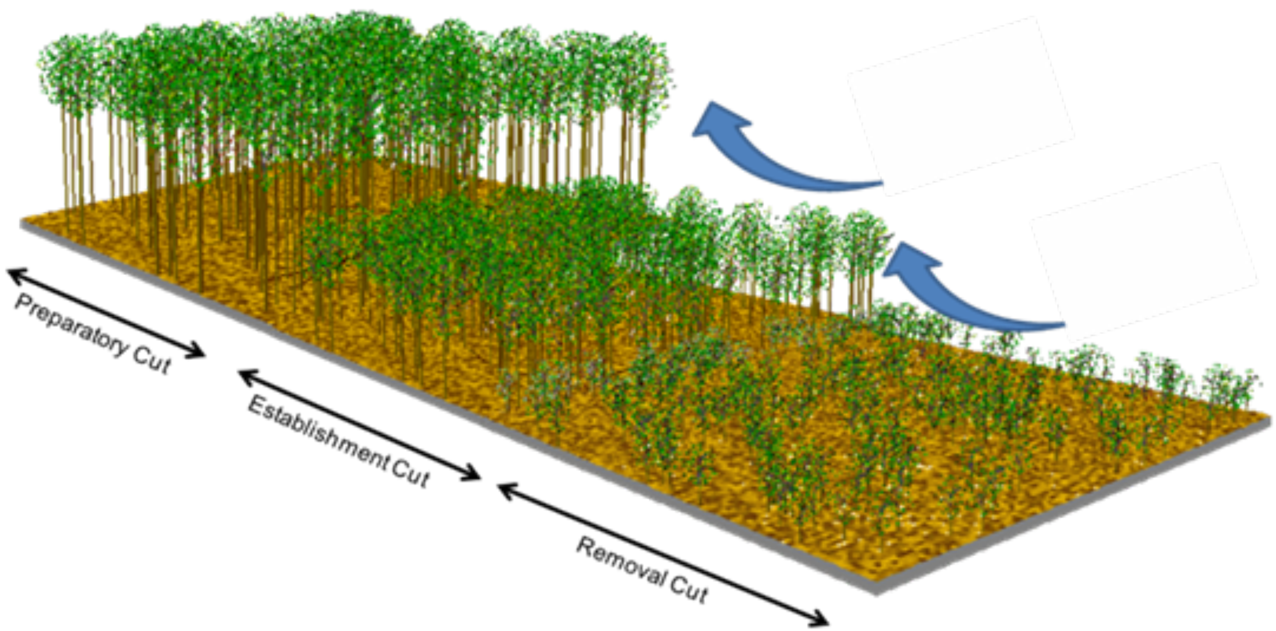
5つのストリップの連続群
の配置

Windstorm Prevention Problem

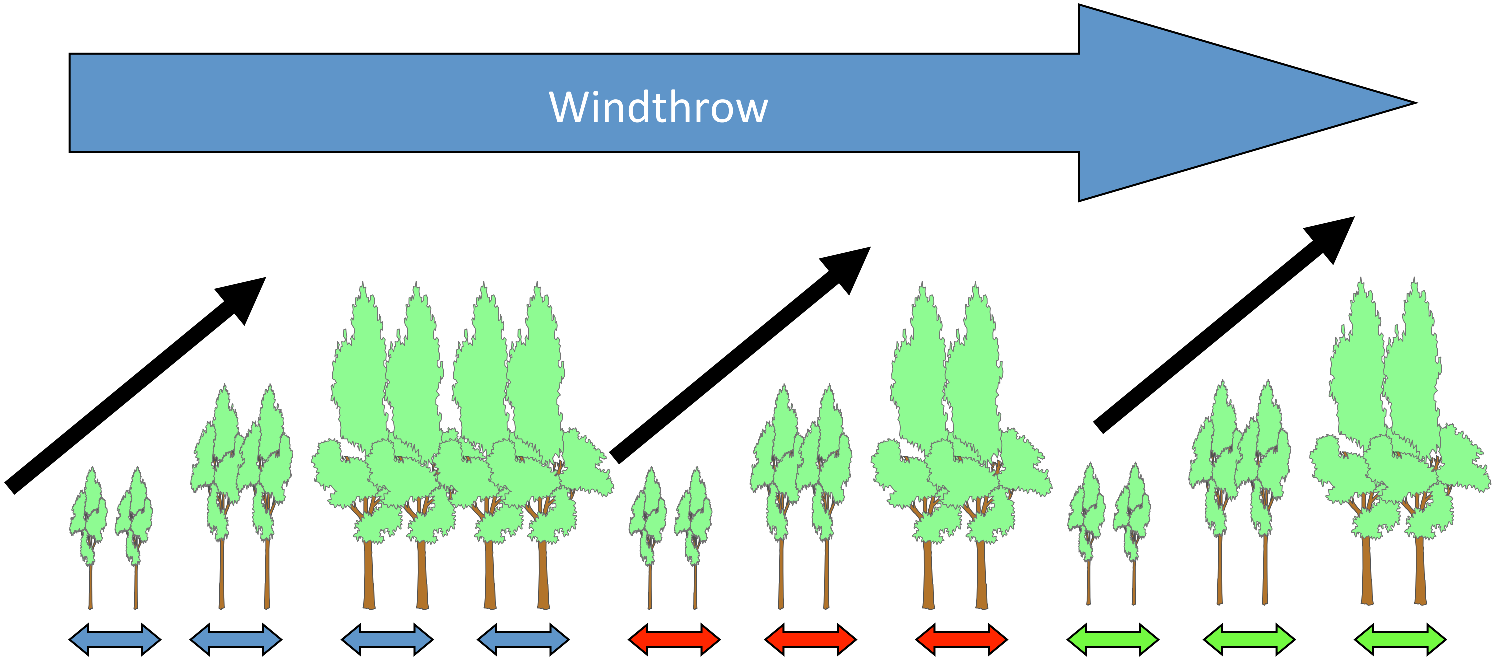
Strip Shelterwood Management
2 cut system



Strip Shelterwood Management
3 cut system



Strip Shelterwood Management Scheme



progress from windward to leeward

Hyper Unit and Neighbors

$NB_i^{(0)}$: **i-th unit itself, 0 degree adjacency**

$NB_i^{(1)}$: **neghbors of the 1st degree adjacency for the i-th unit**

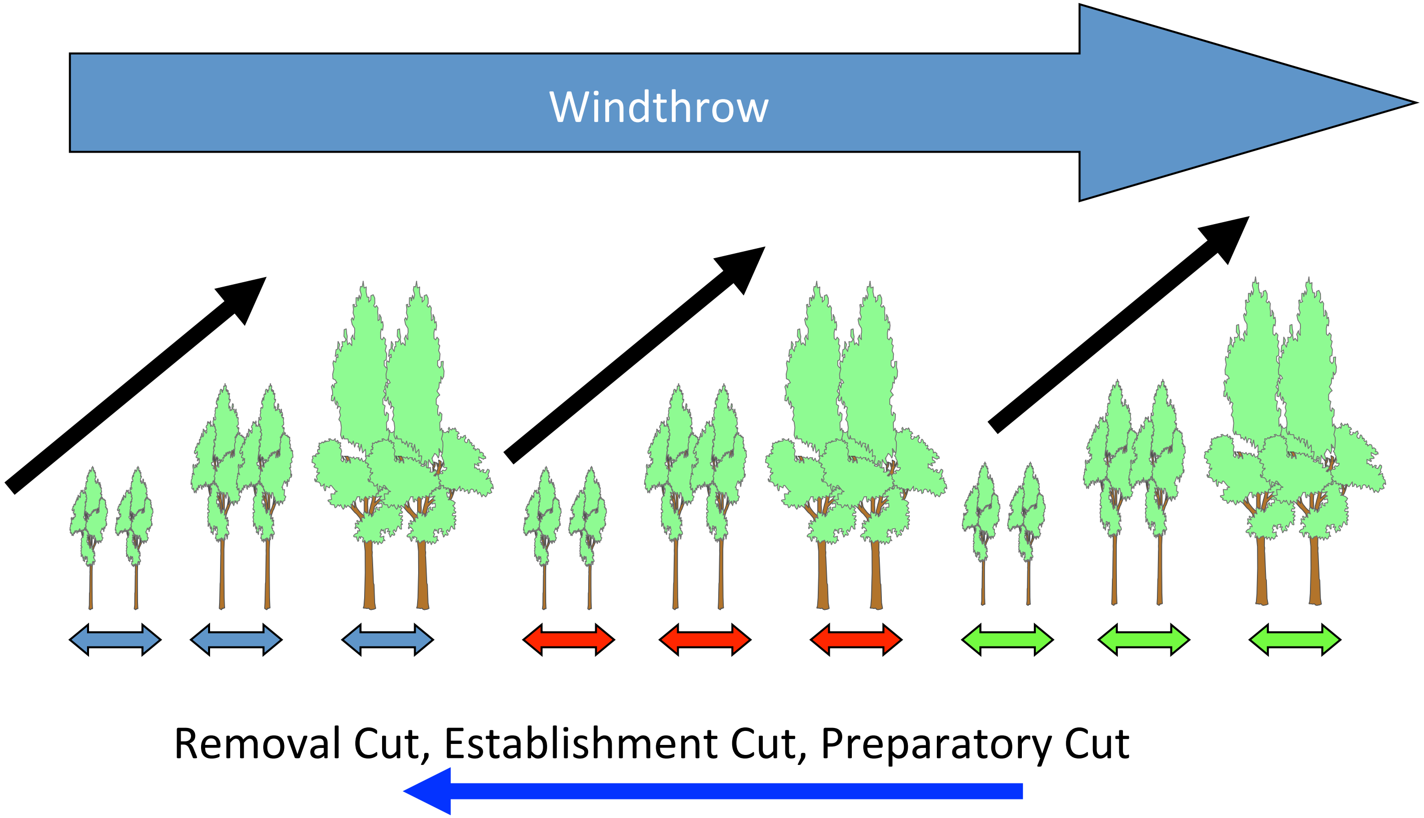
$NB_i^{(2)}$: **neghbors of the 2nd degree adjacency for the i-th unit**

$NB_i^{(n)}$: **neghbors of the n-th degree adjacency for the i-th unit**

$NB_i^{(j)}$: **those units adjacent to** $\{\bigcup_{k=1}^{j-1} NB_i^{(k)}\}$

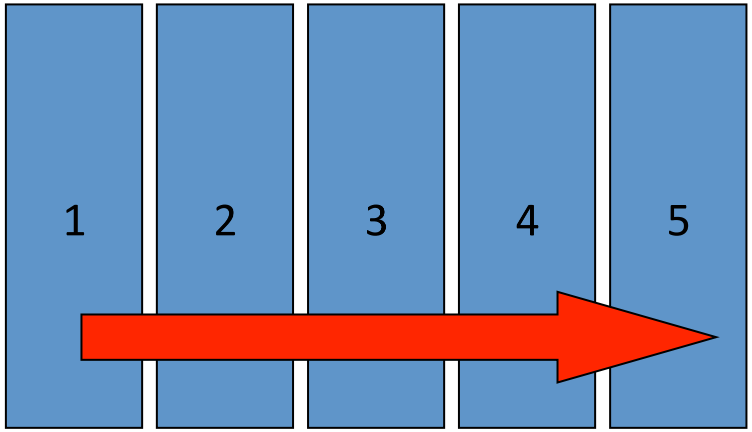
$$HU_i = \bigcup_{k=1}^{M} NB_i^{(k)}$$

Strip Shelterwood Management Scheme



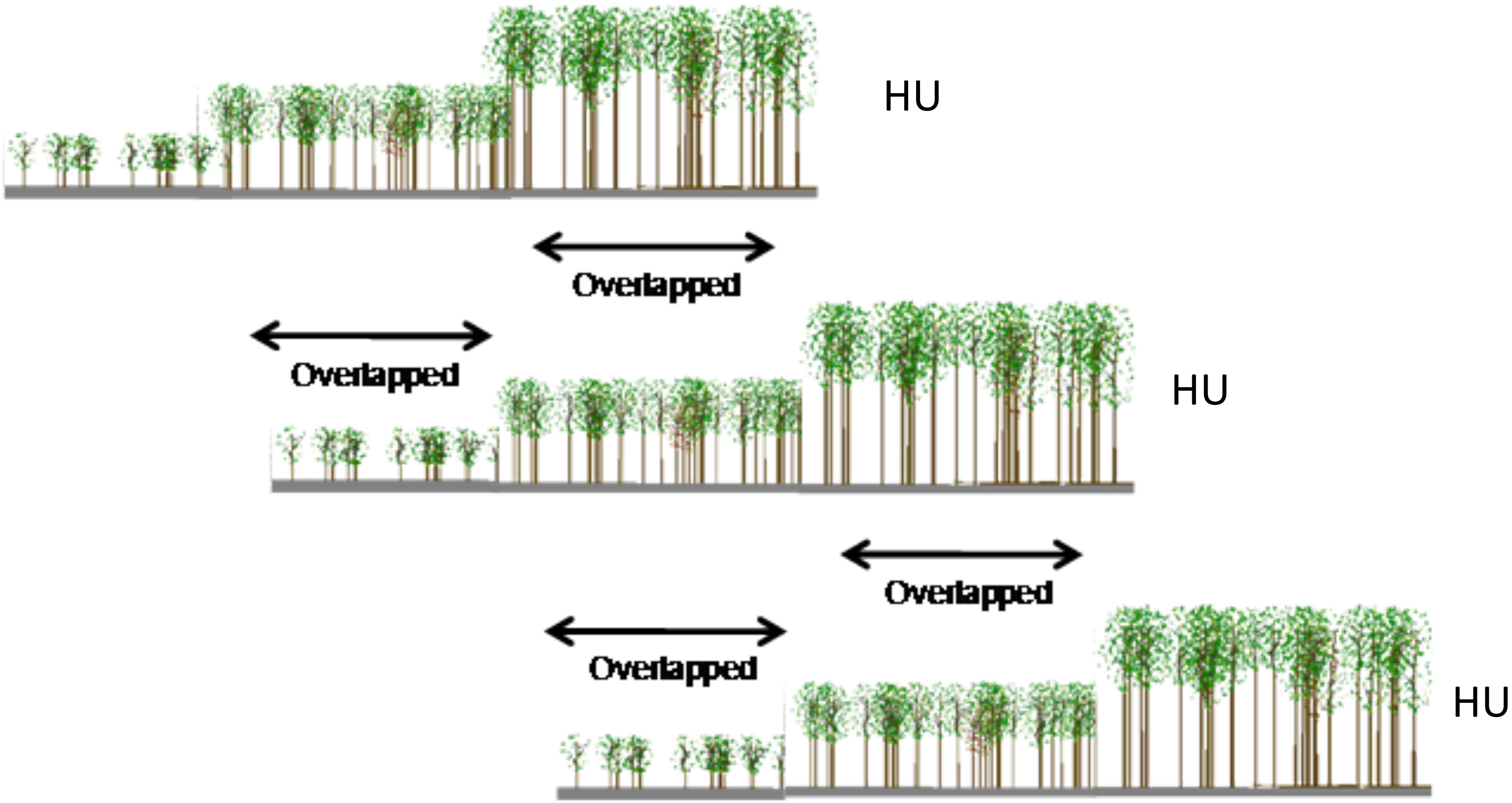
Developing Hyper Unit

- Adjacency for Units from windward to leeward



$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Prevention of overlapping by adjacency HU for 3 cut system



Example for Schelterwood Scheme

Period	Cut	Strip												
		1	2	3	4	5	6	7	8	9	10	11	12	13
	1	S												
1	2	R	S						S					
	3	F	R	S					R	S				
	4		F	R	S				F	R	S			
2	5			F	R	S				F	R	S		
	6				F	R	S				F	R	S	
	7					F	R	S				F	R	S
3	8						F	R					F	R
	9							F						F

S – seeding cut, R – removal cut, F – final cut

Period	Cut	Strip												
		1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	S					S							
	2	F	S				F	S				S		
2	3		F	S				F	S			F	S	
	4			F	S				F	S			F	S
3	5				F	S				F	S			F
	6					F					F			

S – seeding cut, F – final cut

Elements of Hyper Unit

2 Cut System

$$HU_i = \bigcup_{k=1}^{5} NB_i^{(k)}$$

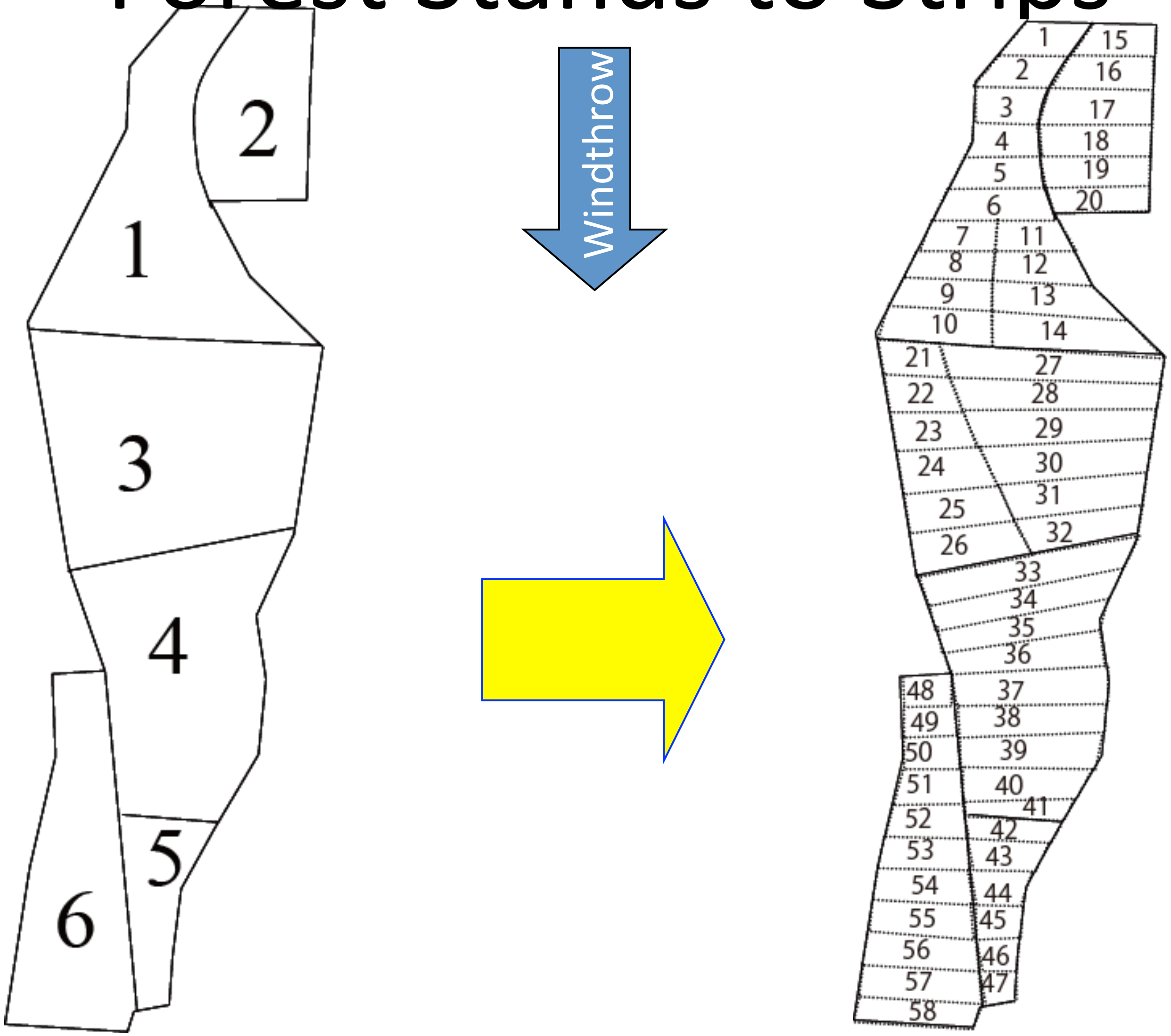
3 Cut System

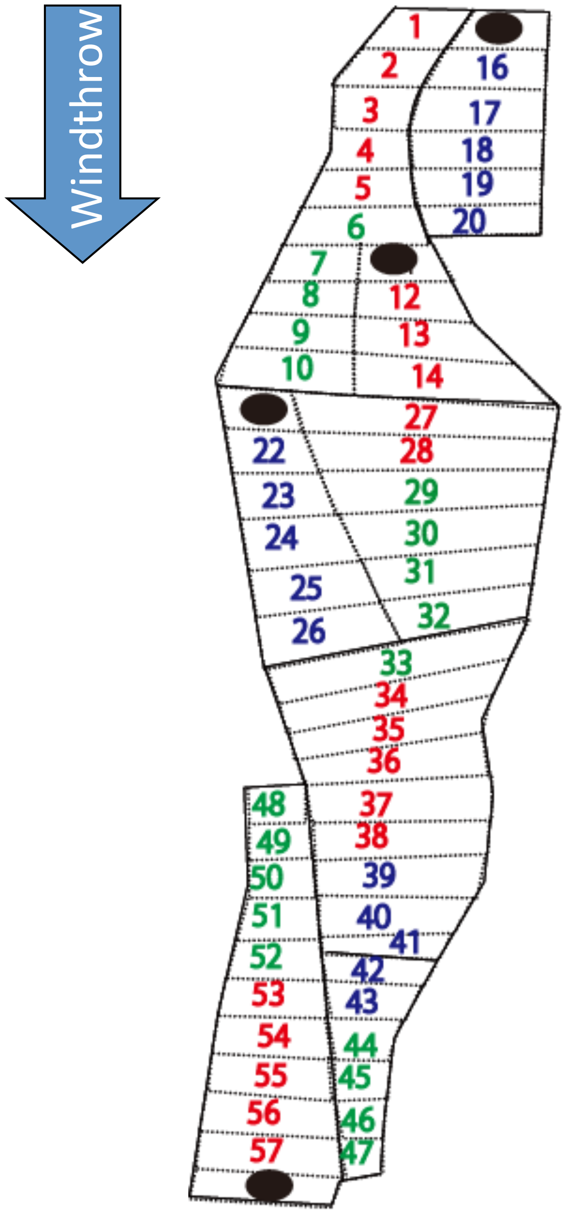
$$HU_i = \bigcup_{k=1}^{7} NB_i^{(k)}$$

Optimization Problem

- Objective: max # of strips to be treated
- Spatial Constraints
  - Avoid overlapping of HUs
  - Avoid the same treatment for adjacent units at the same time (normal adjacency)

Forest Stands to Strips

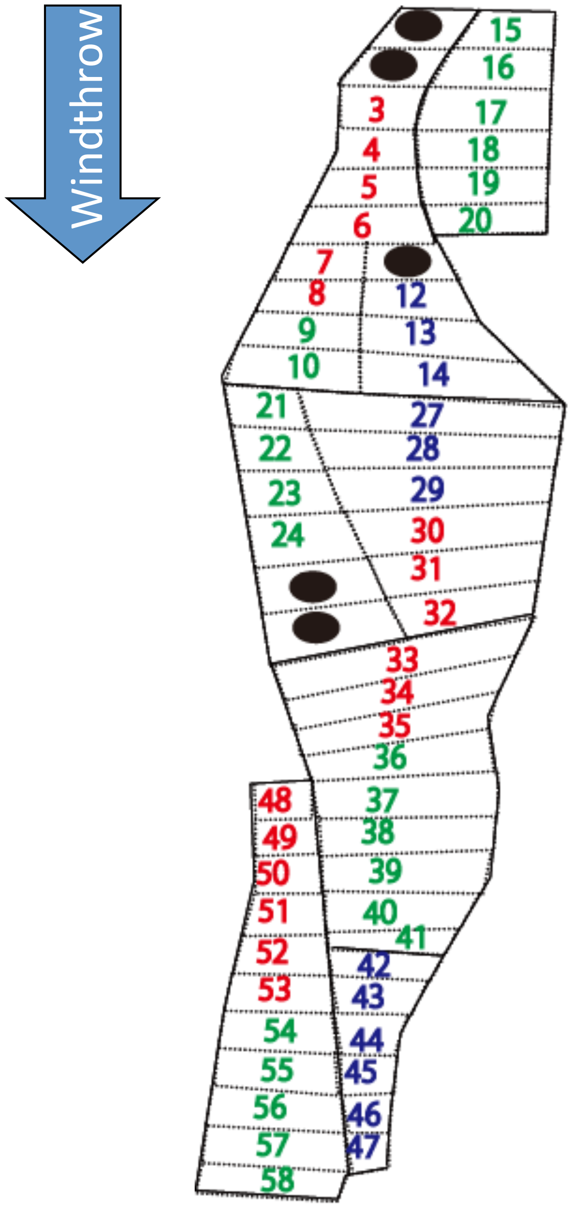




5 Strips Connection

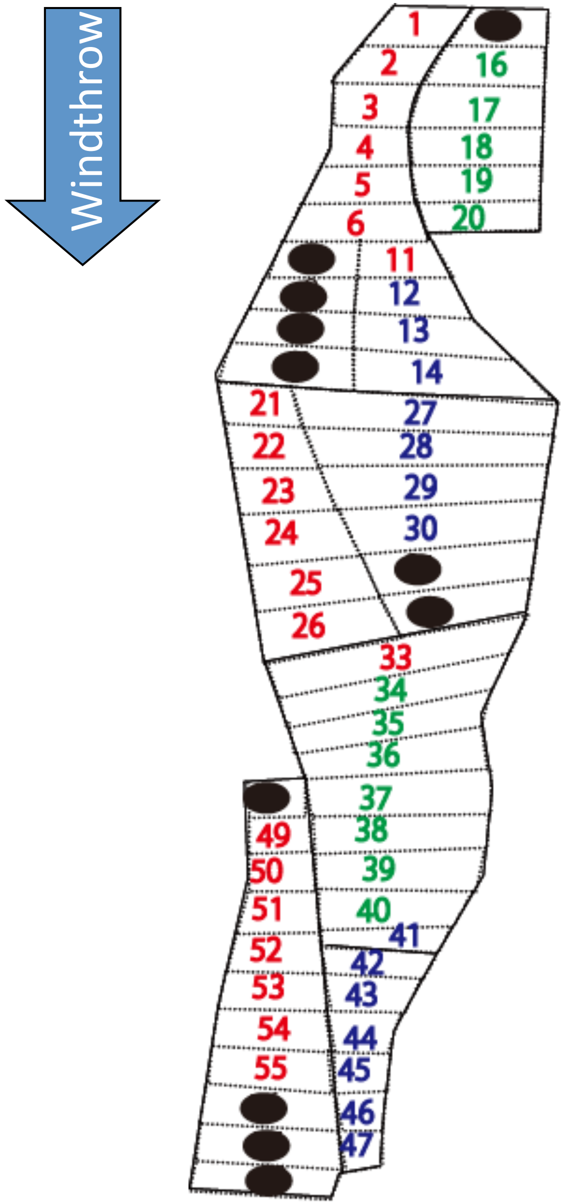
2 Cut System

4 strips uncut



6 Strips Connection

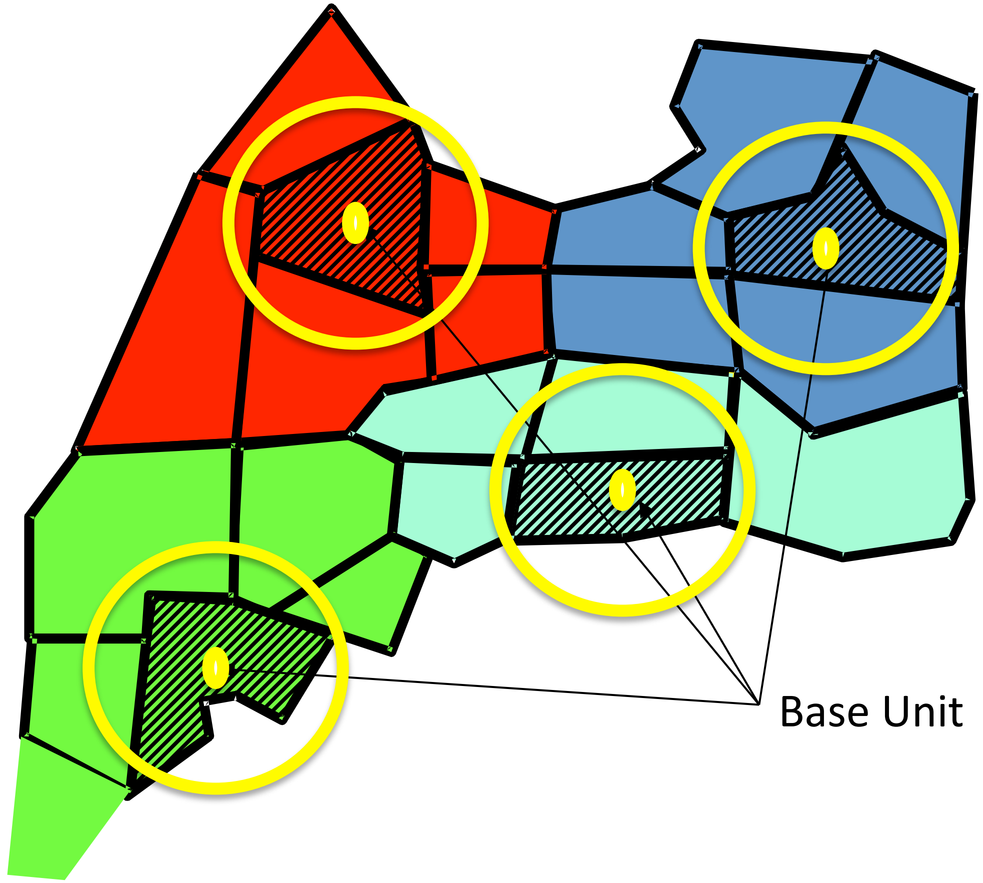
5 strips uncut



7 Strips Connection
3 Cut System
11 strips uncut

Aggregation Problem with Spatial Adjacent Constraints

We assume that one clique is created based on one unit with certain buffer connection



Steps

a) Create candidate cliques (Hyper Units) for aggregation based on units, and define decision variable matrix

Introduce two kinds of index sets:

Original Unit Set: $\mathrm{u}_i = \{i\}$

Hyper Unit Set : $\mathrm{HU}_i = \{\text{indices combined for clique based on } \mathrm{u}_i\}$

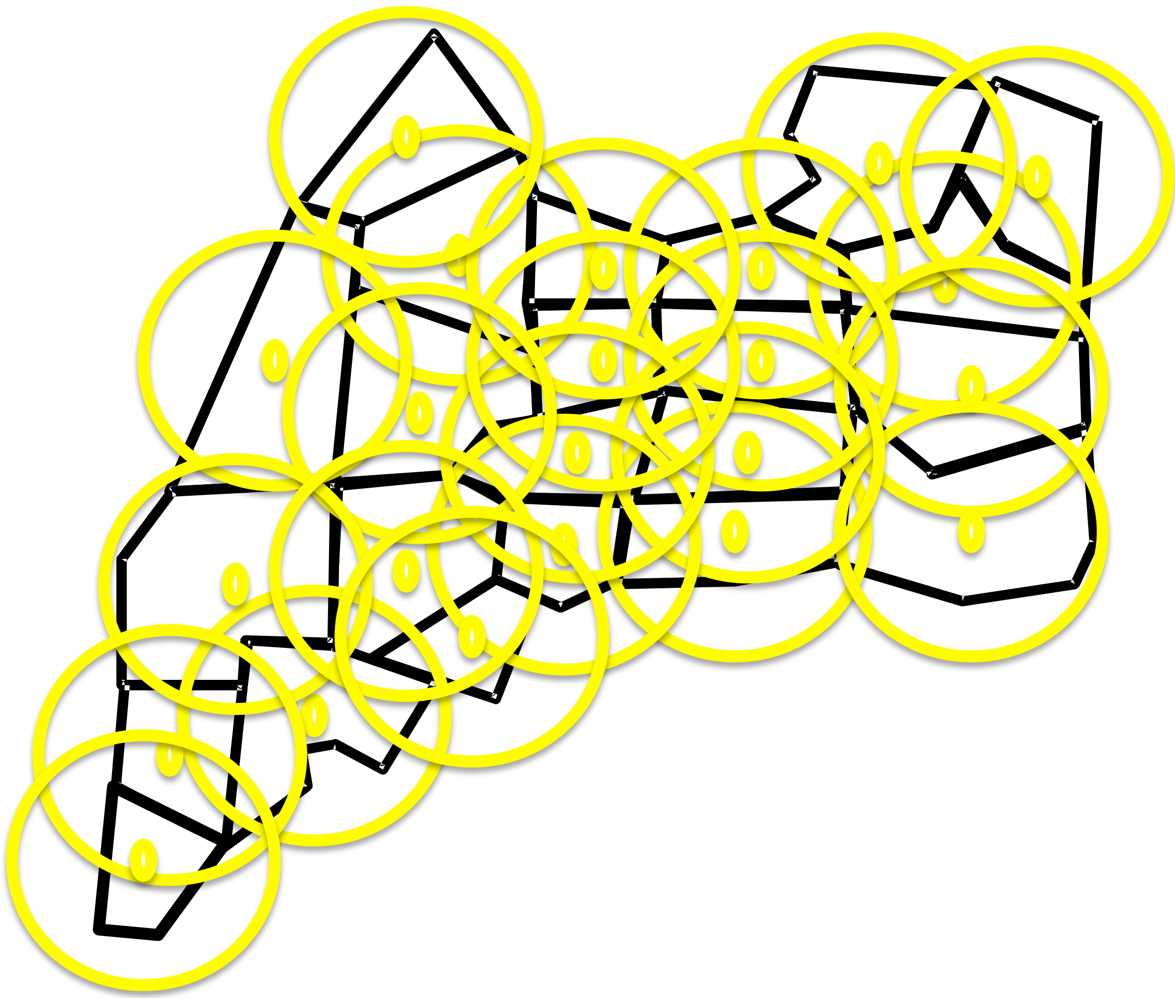
b) Develop extended land accounting constraints for overlapping

c) Develop adjacency constraints for *HU*s and original units

d) Formulate spatially constrained problem with area restriction under multiple harvests

a) Create candidate clique for aggregation based on each unit

Hyper Units $:\mathrm{HU}_i = \{\text{indices combined for clique based on } u_i\}$



Introduce decision variables for *HU*s

$$\boldsymbol{X} \qquad (m \times n) : x_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for } u_i \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{Y} \qquad (m \times n) : y_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for } HU_i \\ 0 & \text{otherwise} \end{cases}$$

Decision Variable Matrix for *HU* and *u*

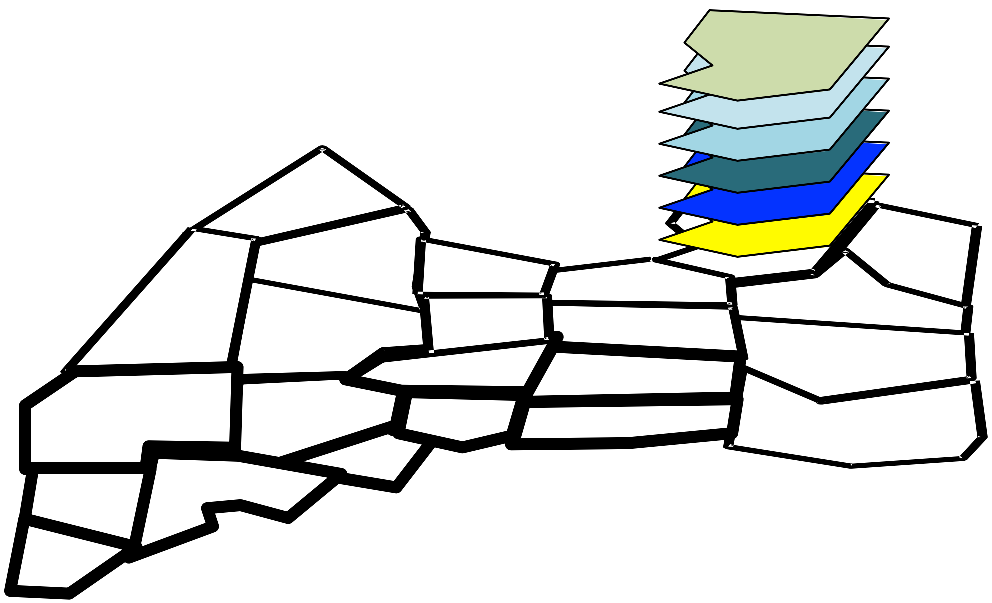
$$\boxed{\boldsymbol{W} = \begin{pmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{pmatrix} \qquad (2m \times n)}$$

b) Develop extended land accounting constraints for overlapping

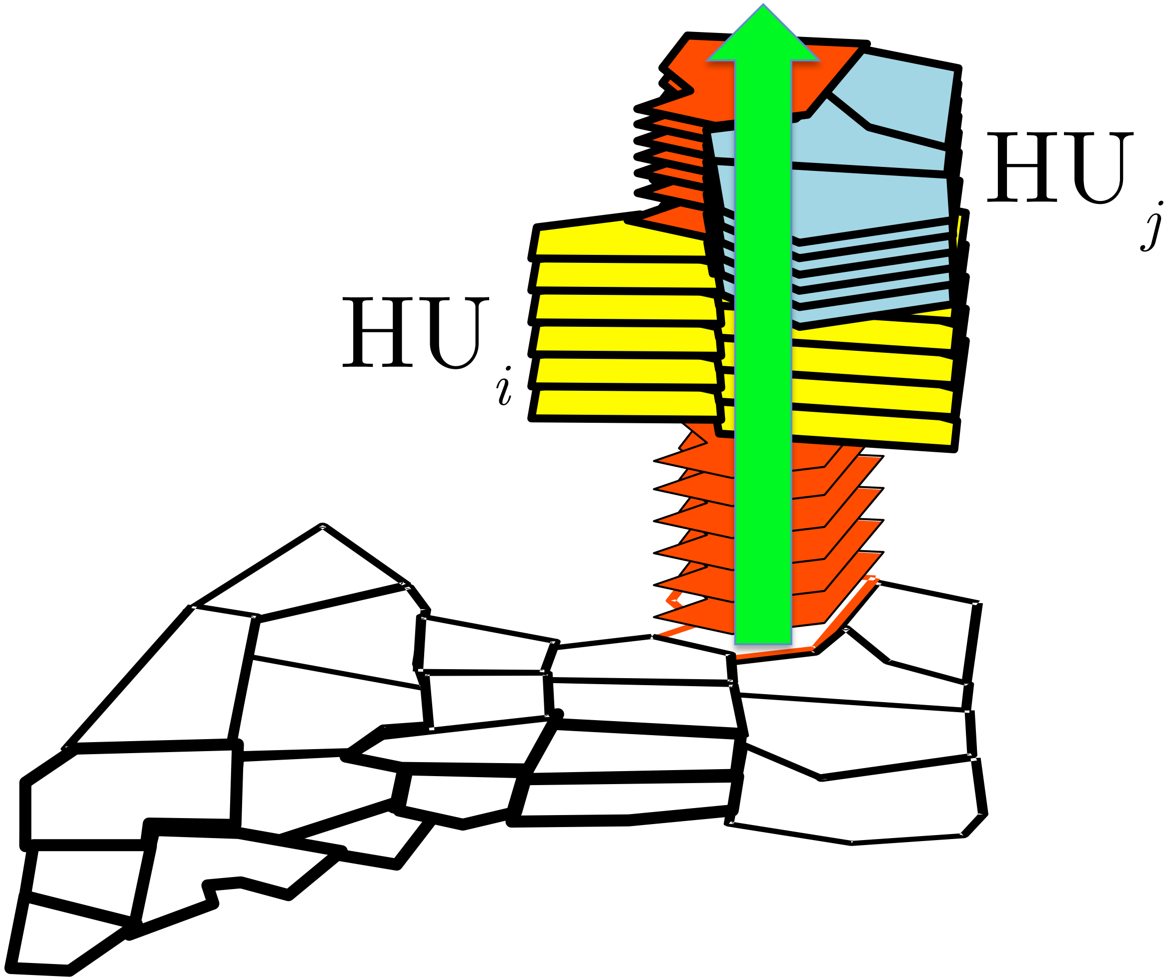
Ordinal land accounting = Overlapping of treatments

$$\boxed{\boldsymbol{X}\mathbf{1}_n \leq \mathbf{1}_m}$$

Different treatments are overlapping for one unit



Different Treatments for *HU* and *u* are overlapping over each unit



Introduce Matrix for Overlapping

For overlapping between u_i and u_j

$$\boldsymbol{A}^{Ou} \quad (m \times m) \qquad : a_{i,j}^{Ou} = \begin{cases} 1 & \text{if } \mathrm{u}_i \cap \mathrm{u}_j \neq \varnothing \\ 0 & \text{if } \mathrm{u}_i \cap \mathrm{u}_j = \varnothing \end{cases}$$

For overlapping between u_i and HU_j

$$\boldsymbol{A}^{Oh} \quad (m \times m) \qquad : a_{i,j}^{Oh} = \begin{cases} 1 & \text{if } \mathrm{u}_i \cap \mathrm{HU}_j \neq \varnothing \\ 0 & \text{if } \mathrm{u}_i \cap \mathrm{HU}_j = \varnothing \end{cases}$$

Overlapping Matrix for *HU* and *u*

$$\boxed{\boldsymbol{A}^{O} = (\boldsymbol{A}^{Ou}, \boldsymbol{A}^{Oh}) : (m \times 2m)}$$

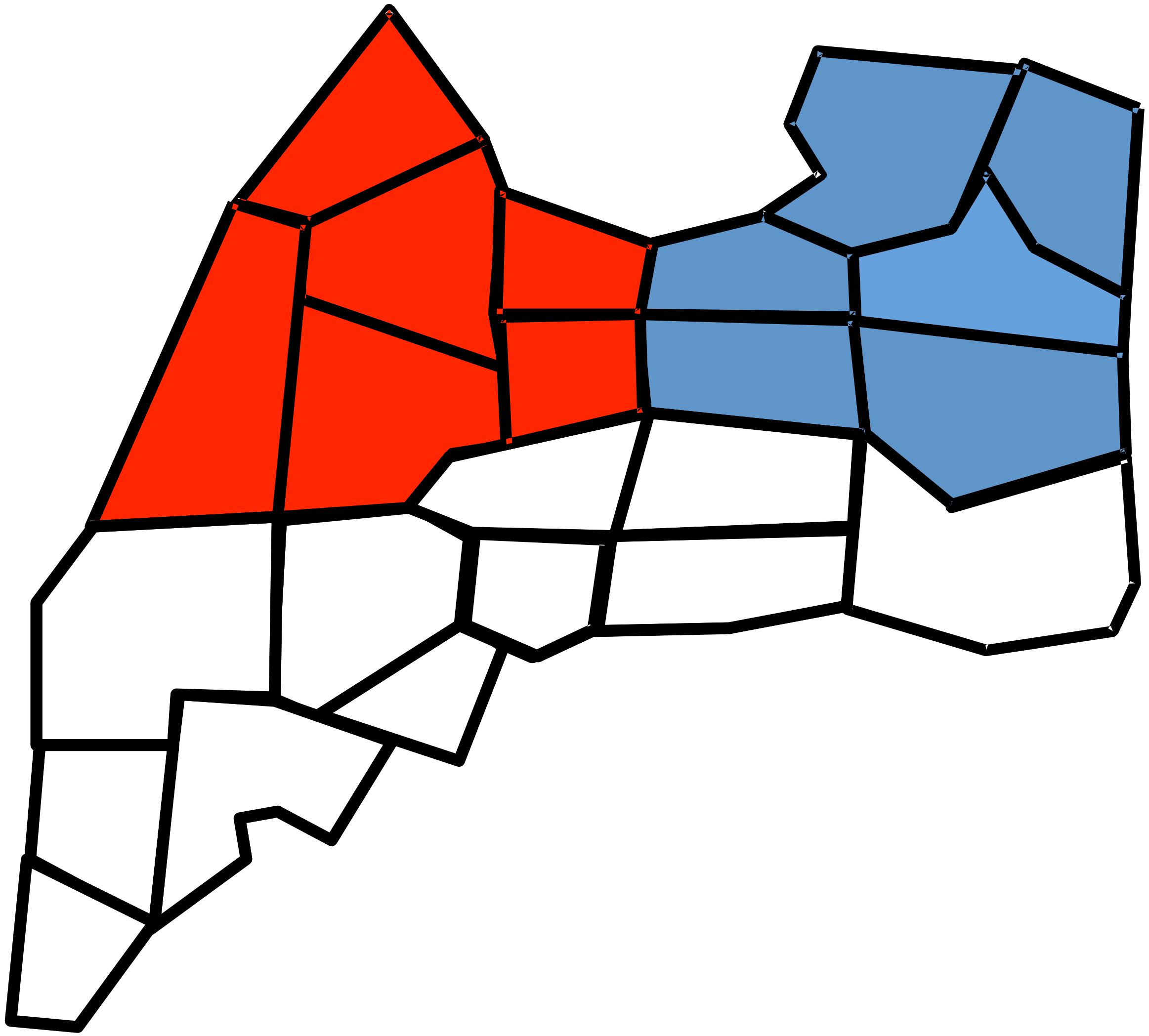
Extended Land Accounting Constraints

$$\boldsymbol{A}^{O} = (\boldsymbol{A}^{Ou}, \boldsymbol{A}^{Oh}) : (m \times 2m)$$

$$\boxed{(\boldsymbol{A}^{O} \otimes \mathbf{1}'_n) \bullet \mathrm{vec}(\boldsymbol{W}') \le \mathbf{1}_m}$$

$$\left(\begin{array}{ccc|c} \multicolumn{3}{c|}{\overbrace{\hphantom{1\cdots1 \quad \cdots \quad 0}}^{\mathrm{u}_i}} & \overbrace{\hphantom{\boldsymbol{A}^{Oh} \otimes \mathbf{1}'_n}}^{\mathrm{HU}_i} \\ 1\cdots1 & \cdots & 0 & \\ \vdots & \ddots & \vdots & \boldsymbol{A}^{Oh} \otimes \mathbf{1}'_n \\ 0 & \cdots & 1\cdots1 & \end{array}\right) \begin{pmatrix} x_{1,1} \\ \vdots \\ x_{m,1} \\ \vdots \\ \hline y_{1,1} \\ \vdots \\ y_{m,n} \end{pmatrix} \le \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

Spatial Adjacency for *HU* and *u*



c) Develop Adjacency Constraints for *HU* and *u*

Hyper Units $\quad :\mathrm{HU}_i = \{\text{indices combined for clique based on } \mathrm{u}_i\}$

+

$$\mathrm{NB}_i = \{\text{index set of units adjacent to the } i\text{-th unit}\}$$

Spatial Adjacency for *HU* and *u*

$$\boldsymbol{A}^S = \left(\begin{array}{c|c} \text{u to u} & \text{u to HU} \\ \hline \text{HU to u} & \text{HU to HU} \end{array}\right) \quad (2m \times 2m)$$

adjacency constraints for *HU* and *u*

Spatial Adjacency for *HU* and *u*

$$\boldsymbol{A}^{S} = \left(\begin{array}{c|c} \boldsymbol{A}_{\text{u-u}} & \boldsymbol{A}_{\text{u-HU}} \\ \hline \boldsymbol{A}_{\text{HU-u}} & \boldsymbol{A}_{\text{HU-HU}} \end{array} \right) \quad (2m \times 2m)$$

$$\boxed{\boldsymbol{A}_{\text{u-HU}} \ (= \boldsymbol{A}'_{\text{HU-u}})}$$

$$\boldsymbol{A}_{\text{u-HU}} : a_{i,j}^{\text{u-HU}} = \begin{cases} 1 & \text{if } \mathrm{u}_i \in \bigcup_{k \in \mathrm{HU}_j} \mathrm{NB}_k \text{ and } \mathrm{u}_i \cap \mathrm{HU}_j = \varnothing \\ 0 & \text{otherwise} \end{cases}$$

$$\boldsymbol{A}_{\text{HU-HU}} : a_{i,j}^{\text{HU-HU}} = \begin{cases} 1 & \text{if } \mathrm{HU}_i \cap \bigcup_{k \in \mathrm{HU}_j} \mathrm{NB}_k \neq \varnothing \text{ and } \mathrm{HU}_i \cap \mathrm{HU}_j = \varnothing \\ 0 & \text{otherwise} \end{cases}$$

Adjacency Constraints for *HU* & *u*

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}^S \otimes \boldsymbol{A}^V \qquad (2mn \times 2mn)$$

$$+$$

$$\boldsymbol{W} = \begin{pmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{pmatrix} \qquad (2m \times n)$$

$$\Downarrow$$

$$[\tilde{\boldsymbol{A}} + \mathrm{diag}(\tilde{\boldsymbol{A}} \cdot \mathbf{1}_{2mn})] \cdot \mathrm{vec}(\boldsymbol{W}') \leq \tilde{\boldsymbol{A}} \cdot \mathbf{1}_{2mn}$$

Summarize: Definition of Decision variable Matrix

Decision variable matrix

$$\boldsymbol{W} = \begin{pmatrix} \boldsymbol{X} \\ \boldsymbol{Y} \end{pmatrix} \qquad \boldsymbol{X} \text{ for a set of } \{\mathrm{u}_i\},\ \ \boldsymbol{Y} \text{ for a set of } \{\mathrm{HU}_i\}$$

$$x_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for } \mathrm{u}_i \\ 0 & \text{otherwise} \end{cases}$$

$$y_{i,j} = \begin{cases} 1 & \text{if the } j\text{-th treatment is implemented for } \mathrm{HU}_i \\ 0 & \text{otherwise} \end{cases}$$

Definition of Coefficient Matrix

Coefficient matrix

$$\tilde{\boldsymbol{C}} = \begin{pmatrix} \boldsymbol{C} \\ \bar{\boldsymbol{C}} \end{pmatrix} \qquad (2m \times n)$$

$c_{i,j}$: coefficient of $x_{i,j}$, $\overline{c}_{i,j}$: coefficient of $y_{i,j}$

$$\overline{c}_{i,j} = (1+\alpha) \sum_{k \in \mathrm{HU}_i} c_{k,j}, \ \alpha : \ \text{reward rate}$$

Definition of Volume Matrix

Volume flow matrix at period p

$$\tilde{\boldsymbol{V}}_p = \begin{pmatrix} \boldsymbol{V}_p \\ \bar{\boldsymbol{V}}_p \end{pmatrix} = \left(\begin{array}{ccc} v^p_{1,1} & \cdots & v^p_{1,n} \\ \vdots & \ddots & \vdots \\ v^p_{1,m} & \cdots & v^p_{m,n} \\ \hdashline \overline{v}^p_{1,1} & \cdots & \overline{v}^p_{1,n} \\ \vdots & \ddots & \vdots \\ \overline{v}^p_{1,m} & \cdots & \overline{v}^p_{m,n} \end{array}\right) \qquad (2m \times n)$$

$v^p_{i,j}$: volume flow from $x_{i.j}$, $\overline{v}^p_{i,j}$: volume flow from $y_{i.j}$

$$\overline{v}^p_{i,j} = \sum_{k \in \mathrm{HU}_i} v^p_{k,j}$$

d) Formulation with area restriction under multiple harvests

Max PNV

$$Z = \max_{\boldsymbol{X}} \operatorname{tr}(\tilde{\boldsymbol{C}}'\boldsymbol{W}) = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{i,j} \cdot x_{i,j} + \overline{c}_{i,j} \cdot y_{i,j})$$

$st.$

$(\boldsymbol{A}^{O} \otimes \boldsymbol{1}_{n}') \bullet \operatorname{vec}(\boldsymbol{W}') \leq \boldsymbol{1}_{m}$ Extended Land Accounting Constraints

$(1-\alpha)v_{0} \leq \operatorname{tr}(\tilde{\boldsymbol{V}}_{p}'\boldsymbol{W}) \leq (1+\alpha)v_{0}, \quad p = 1, 2, \cdots, T$

Harvest Flow Constraints

$[\tilde{\boldsymbol{A}} + \operatorname{diag}(\tilde{\boldsymbol{A}} \cdot \boldsymbol{1}_{mn})] \cdot \operatorname{vec}(\boldsymbol{W}') \leq \tilde{\boldsymbol{A}} \cdot \boldsymbol{1}_{mn}$

$\tilde{\boldsymbol{A}} = \boldsymbol{A}^{S} \otimes \boldsymbol{A}^{V}$ Adjacency Constraints